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ANALYSIS OF THE POWER REQUIREMENT OF A BLOWING AIRFOIL WITH SLOTTED FLAP

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FAIRCHILD AIRCRAFT DIVISION
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SUBJECT

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FOREWORD

The work of this report has been done in partial fulfillment of Contract Nonr 2226(00) between the Air Branch, Office of Naval Research and the Fairchild Aircraft Division.

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SUMMARY

If the lift of an airfoil shall be increased by blowing a jet over the flap, power must be expended to create initial amounts of jet momentum flux and jet stiffness at the jet exit (blowing slot). Skin friction, turbulent mixing, and pressure variations along the flap cause considerable changes of the jet properties from blowing slot to trailing edge. In the present experiments the jet momentum flux was always decreased whereas the jet stiffness was increased under certain favorable conditions. The total lift consists of the basic lift, of the vertical component of the reaction of the total momentum flux at the trailing edge and of the jet-induced lift. The jet-induced lift results from the vorticity of the mixing jet sheet downstream of the trailing edge and from the sink effect of the mixing jet along the flap. Because of mixing the stiffness of the jet sheet increases downstream from the trailing edge toward infinity. Thus, the contribution of jet-sheet vorticity to the jet-induced lift becomes greater than predicted by the theory of the blowing wing with non-mixing jet. The analysis was restricted to states of flow where the flow was fully attached to the flap. Under these conditions losses of wing-circulation lift are caused by leading-edge separation. Reattachment of the flow separated at the leading edge is, however, favored by the sink effect of the mixing jet on the pressure distribution over the suction side of the main airfoil, resulting in an alleviation of the losses of wing-circulation lift. The influences of flap-deflection angle, angle of attack and blowing-slot width ratio on the lift-power relationship are studied and represented by empirical formulae permitting extrapolation of this relationship over the range of power coefficients attainable in the present experiments.

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING

1. The Two-Dimensional Blowing Wing in Non-Viscous Incompressible Flow

In order to simplify the mathematical problem, the theory of the blowing wing has been based on a set of simplifying assumptions about the blowing jet and the airfoil, namely: thin jet, no jet mixing, constant pressure along the jet sheet, constant total pressure within the jet sheet, thin airfoil, small angles of attack and deflection, etc. Under these conditions, the pressure discontinuity across the jet sheet which is required to overcome the excess centrifugal forces of the jet sheet and to bend the jet sheet into the curved pattern of the surrounding basic flow (Ref. 1) becomes

$$\Delta p_j = \rho (V_j^2 - V_o^2) \frac{h}{r}. \qquad (1)$$

Here ρ is the density, V_j the jet velocity, V_o the free-stream velocity, h the thickness of the jet sheet and r its local radius of curvature. The pressure discontinuity is accompanied by a discontinuity of the velocities across the jet sheet, ε, determined by

$$\Delta p_j = \rho V_o \varepsilon. \qquad (2)$$

The discontinuity ε is the lifting vorticity of the jet sheet which causes the jet-induced lift on the airfoil (Ref. 1).

Dividing Δp_j by the free-stream dynamic pressure, $q_o = \rho \frac{V_o^2}{2}$, we obtain

$$\frac{\Delta p_j}{q_o} = 2 \frac{\varepsilon}{V_o} = 2 \frac{V_j^2 - V_o^2}{V_o^2} \frac{h}{r} = 2 \frac{\lambda_c}{c} \frac{c}{r} = C_E \frac{c}{r}. \qquad (3)$$

Ref. 1: H.B. Helmbold, University of Wichita, Engineering Study 110 (August 1953)

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

The characteristic length $\lambda_c \equiv [(V_j/V_o)^2 - 1]h$ which, according to our assumptions, is constant along the jet sheet is called the centrifugal thickness of the jet sheet, and the nondimensional quotient $C_E \equiv 2\lambda_c/c$, where c denotes the airfoil chord, is called the stiffness coefficient of the jet sheet. Equation (3) states that the nondimensional resistance to bending of the jet sheet, $\Delta p_j/q_o$, is the product of the stiffness coefficient and the nondimensional curvature of the jet sheet, c/r.

The total lift coefficient of the blowing airfoil may be resolved into three terms,

$$C_L = C_{lo} + \Delta C_{l\Gamma} + C_{lM}. \tag{4}$$

The first term is the basic lift coefficient of the airfoil without blowing jet, the second term is the jet-induced lift coefficient, the third term represents the vertical reaction of the blowing jet. The jet-induced lift coefficient is a function of the stiffness coefficient. The sum of the two first terms is called the wing-circulation lift coefficient,

$$C_{l\Gamma} = C_{lo} + \Delta C_{l\Gamma}. \tag{5}$$

The last term of Equation (4), the momentum-lift coefficient, is

$$C_{lM} = C_\mu \sin \delta_{te} \tag{6}$$

if the blowing air is taken from the interior of the airfoil and not from the freestream; such is the case with a two-dimensional airfoil model in a wind tunnel where the blowing air

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

is supplied from outside the tunnel. The jet-momentum coefficient is defined by

$$C_\mu = \frac{m_j V_j}{q_o bc} = \frac{\rho_j V_j^2 bs}{q_o bc} = 2 \frac{\rho_j}{\rho_o} \left(\frac{V_j}{V_o}\right)^2 \frac{s}{c} \qquad (7)$$

where $s = h$ is the width of the blowing slot, b the infinite span, m_j the jet-mass flux. The inclination of the jet sheet, at the trailing edge, toward the free stream velocity is denoted by the jet-deflection angle δ_{te}.

2. The Mixing Jet under Constant Pressure

If the jet is blown over a deflected flap, as in the experiments to be discussed later on, it undergoes violent pressure variations until it reaches the trailing edge. This process presents a problem difficult for two reasons: the mechanism of turbulent mixing under variable pressure is virtually unknown, and the compound of differential equations governing the process is rather complicated even if the model of the mixing mechanism is idealized by the similarity hypothesis (Ref. 2). Under such a state of affairs it will be instructive to study the simpler case of the mixing jet under constant pressure, where, fortunately, certain statements can be made without any detailed knowledge of the mixing mechanism.

Let us consider the mixing jet in a parallel stream V_o. The mass flux between two streamlines symmetrical to the jet axis and situated outside the mixing zone is

$$m = b\left(\rho \int^h u \cdot dy + \rho V_o a\right) = \text{const} \qquad (8)$$

Ref. 2: H.B. Helmbold, University of Wichita, Engineering Study 182, (August 1955)

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

where h is the width of the mixing zone and h + a the distance between the symmetrical streamlines; u is the local velocity in the mixing jet. The momentum flux between the same streamlines is constant too,

$$M = b\left(\rho \int^h u^2 \cdot dy + \rho V_o^2 a\right) = \text{const}, \tag{9}$$

because with constant pressure no pressure forces are acting on the flow. From (8) and (9) follows that the excess-momentum flux, ΔM, is also a constant,

$$\Delta M \equiv M - mV_o = b\rho \int^h (u - V_o)\, u \cdot dy = \text{const}. \tag{10}$$

Distinctive from m and M, which are properties of the flow between arbitrarily chosen streamlines, the excess-momentum flux ΔM is a property of the mixing jet itself. Therefore, it is useful to define a characteristic length of the mixing jet, its excess-momentum thickness, by

$$\lambda \equiv \frac{\Delta M}{\rho_o V_o^2 b} = \int^h \frac{(u - V_o)\, u}{V_o^2}\, dy = \frac{\rho_j}{\rho_o}\frac{(V_j - V_o)V_j}{V_o^2}\, s. \tag{11}$$

The last form refers to the state of flow in the jet nozzle. The length λ is connected with more conventional notions by forming the nondimensional quotient

$$\frac{\lambda}{c} = \frac{\rho_j}{\rho_o}\frac{(V_j - V_o)V_j}{V_o^2}\frac{s}{c} = \frac{C_\mu}{2} - C_Q = \frac{C_\mu}{2} - \sqrt{\frac{C_\mu}{2}\frac{\rho_j}{\rho_o}\frac{s}{c}}. \tag{12}$$

Here

$$C_Q \equiv \frac{m_j}{\rho_o V_o bc} = \frac{\rho_j}{\rho_o}\frac{V_j}{V_o}\frac{s}{c} \tag{13}$$

is the jet-mass coefficient (quantity coefficient). By comparison with (7),

$$C_Q = \sqrt{\frac{C_\mu}{2}\frac{\rho_j}{\rho_o}\frac{s}{c}}. \tag{14}$$

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

If the jet air is taken from the free stream the jet thrust is equal to the excess-momentum flux. On the other hand, if the jet air is supplied by a source within the airfoil, the jet-mass flux has first to be accelerated to free-stream velocity which produces an additional thrust, $m_j V_o$. Hence, in this case, the total jet thrust becomes

$$T_j = b\rho \int^h (u - V_o)\, u \,.\, dy + \rho_j V_j bs. V_o$$

$$= \rho_j (V_j - V_o)\, V_j bs + \rho_j V_j V_o bs = \rho_j V_j^2 bs = m_j V_j . \qquad (15)$$

This means that under constant pressure the jet thrust equals the jet-momentum flux at the nozzle exit. (Of course, this result is strictly valid only if no outer forces, like frictional forces along a solid surface adjacent to the jet, act on the jet). Mixing by itself does not affect the jet-momentum flux.

With the mixing jet the centrifugal thickness is defined by

$$\lambda_c \equiv \int^h \frac{u^2 - V'^2}{V_o^2}\, dy \qquad (16)$$

where $$V'^2 = 2\,\frac{p_{to} - p}{\rho_o} \qquad (17)$$

with p_{to} denoting the free-stream total pressure and p the local static pressure. Under constant pressure, $p = p_o$, $V' = V_o$, the centrifugal thickness becomes

$$\lambda_c = \int^h \frac{u^2 - V_o^2}{V_o^2}\, dy = \int^h \frac{u + V_o}{u}\, \frac{(u - V_o)u}{V_o^2}\, dy. \qquad (18)$$

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

At the nozzle exit, $h = s$, $u = V_j$, and if the slot is very narrow, $s \to 0$, $u/V_o \to \infty$ and $(u + V_o)/u \to 1$. Hence

$$\lambda_c \to \int^h \frac{(u-V_o)\, u}{V_o^2}\, dy = \lambda \text{ in a narrow blowing slot.} \tag{19}$$

At infinity downstream, $u/V_o \to 1$ and $(u + V_o)/u \to 2$. Hence

$$\lambda_c \to 2 \int^h \frac{(u-V_o)\, u}{V_o^2}\, dy = 2\lambda \text{ at infinity downstream.} \tag{20}$$

Since $C_E = 2\lambda_c/c$, this discussion shows that the stiffness coefficient of a mixing jet under constant pressure increases from blowing slot to infinity downstream by a factor 2, approximately.

If the mechanism of turbulent mixing is idealized by the similarity hypothesis the ratio λ_c/λ can be computed as a function of the nondimensional distance, x/λ, from the infinitely narrow blowing slot (Ref. 3). This function is presented in Fig. 1 where λ_c/λ is plotted against $\sqrt{x/\lambda}$. A numerical example may illustrate the application of this diagram. If $C_\mu = 1.16$ and $s/c = 0.009$ are the given data Eq. (12) yields $\lambda/c = 0.5077$. At the jet nozzle $\lambda_c/c = \left[(V_j/V_o)^2 - 1\right]\ s/c = (C_\mu/2) - (s/c) = 0.571$. Hence, at the same station, $\lambda_c/\lambda = 1.125$. The downstream distance of the jet nozzle from the equivalent infinitely narrow slot is read from Fig. 1, namely $x/\lambda = 0.250^2 = 0.0625$. Let us asume that the trailing edge is at the downstream distance $\Delta s/c = 0.22$ or $\Delta x/\lambda = 0.433$ from the jet nozzle. This is a downstream distance $x/\lambda = 0.496 = 0.705^2$ from the infinitely narrow slot. For this station a value $\lambda_c/\lambda = 1.298$ is read from Fig. 1. The increase of the centrifugal thickness from jet nozzle to trailing edge is 15.4%, and its increase from the trailing edge to infinity downstream is 54%.

Ref. 3: H.B. Helmbold, University of Wichita, Engineering Study 137 (May 1954); now contained in Engineering Study 294.

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

The increase from jet nozzle to infinity downstream is 77.8%. Since the jet-induced lift is an integral over the induction effect of the product of stiffness coefficient and nondimensional curvature of the jet sheet behind the airfoil, the stiffness coefficient being $C_E = 2\lambda_c/\lambda$, any theory basing the lift computation on the stiffness coefficient at the jet nozzle or even on its value at the trailing edge must underestimate the jet-induced lift.

3. The Two-Dimensional Blowing Wing in Viscous Incompressible Flow.

With a mixing jet another contribution to the jet-induced lift can be produced by the entrainment of the surrounding potential flow into the mixing zone. The effect of the entrainment on the potential flow may be considered as a sink effect.

Let us compare the mass fluxes between the two streamlines of Section A2, for the physical flow and for a fictitious flow where $u = V' =$ const over a cross-section and V' is defined by Eq. (17). With the physical flow the mass flux is

$$m = b\,(\rho \int^{h} u\,.\,dy + \rho V' a) = \text{const}, \qquad (21) \qquad (21)$$

cf. Eq. (8).

With the fictitious flow the mass flux is

$$m' = b\,\rho V'\,(h+a) = m - b\rho \int^{h} (u-V')\,dy. \qquad (22)$$

This is differentiated with respect to downstream distance,

$$\frac{dm'}{dx} = b\rho\,\frac{d}{dx}\left[V'\,(h+a)\right] = -d\rho\,\frac{d}{dx}\int^{h} (u-V')dy. \qquad (23)$$

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

The mass flux of the fictitious flow shows a downstream variation which can only exist if the flow contains sinks ($dm'/dx < 0$) or sources ($dm'/dx > 0$). The downstream variation dm'/dx is negative unless the velocity gradient

$$\frac{dV'}{dx} = -\frac{dp}{\rho V' \cdot dx} \tag{24}$$

exceeds a certain limit, approximately determined by Eq. (20) of Ref. 2. (The limit is large near the blowing slot, small at far distances downstream.)

If the jet is blown over the rear portion of the suction side of an airfoil the sink distribution induces an additional lift of the same type as the additional lift induced by distributed suction along the suction side (Ref. 4). The simplest case is the flat plate with blowing jet at a small angle of attack. Let x_c be the downstream distance of the blowing slot from the leading edge. Then the additional lift coefficient induced by the sink effect of the mixing jet upstream of the trailing edge is

$$\Delta C_{lS} = -\frac{2}{\rho_o V_o b_c} \int_{x_o}^{c} \frac{dm'}{dx} \sqrt{\frac{x}{c-x}}\, dx. \tag{25}$$

In this special case the contribution of sinks located downstream of the trailing edge is negligible. Generally, their contribution is positive with positive jet deflection angles, $\delta_{te} > 0$. With constant pressure and constant density, ΔC_{lS} is proportional to

$$\frac{V_j - V_o}{V_o} \cdot \frac{s}{c} = C_Q - \frac{s}{c} = \sqrt{\frac{C_\mu}{2} \frac{s}{c}} - \frac{s}{c}. \tag{26}$$

Ref. 4: F. Ehlers, Aerodynamische Versuchsanstalt Göttingen, Bericht 45/W/15 (1945)

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING

It has been shown in Section A2. that certain quantities, namely excess-momentum flux (10) and total momentum flux (15), are constant along the mixing jet under constant pressure unless the mixing zone comes in contact with solid surfaces. For variable pressure these quantities have to be redefined as follows:

The excess-momentum flux (10) becomes

$$\Delta M = b \int^{h} \rho \, (u - V') \, u . dy \tag{27}$$

which is no longer constant along the mixing jet. The total momentum flux becomes

$$J = b \int^{h} \rho \, (u - V') \, u . dy + \rho_j V_j V' bs \tag{28}$$

which is also no longer constant. In the blowing slot where $\rho = \rho_j$, $u = V_j$, $h = s$, Equation (28) specialises to

$$J = J_{slot} = \rho_j V_j^2 \, bs = m_j V_j . \tag{29}$$

This is the jet-momentum flux. Its nondimensional coefficient has been defined by Eq. (7). In order to avoid confusion we will use the subscript "slot" if it becomes necessary: $C_\mu = (C_\mu)_{slot}$. The same notation with the subscript "te", standing for "trailing edge," is used for the nondimensional coefficient of the total momentum flux, Eq. (28), at the trailing edge,

$$(C_\mu)_{te} = \frac{J}{q_o bc} = 2 \left(\int^{h/c} \frac{\rho}{\rho_o} \frac{(u - V')u}{V_o^2} d\frac{y}{c} + \frac{\rho_j}{\rho_o} \frac{V_j V'}{V_o^2} \frac{s}{c} \right)_{te} \tag{30}$$

where $\rho/\rho_o = 1$ for incompressible flow. Since V', as defined by Eq. (17), varies across the mixing jet an average value V'_{av}

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING

has to be inserted in the second term within the parenthesis of Eq. (30). In our evaluations the value V' at that station, where the excess total pressure Δp_t has its maximum, has been substituted as an approximation to the correct average.

The stiffness coefficient at the trailing edge is

$$(C_E)_{te} = 2 \frac{\lambda_c}{c} = 2 \int^{h/c} \frac{\rho u^2 - \rho_o V'^2}{\rho_o V_o^2} d \frac{y}{c}. \qquad (31)$$

For incompressible flow, $\rho/\rho_o \doteq 1$, the Bernoulli law yields

$$(C_E)_{te} = 2 \int^{h/c} \frac{\Delta p_t}{q_o} d \frac{y}{c} . \qquad (32)$$

In the blowing slot the stiffness coefficient is

$$(C_E)_{slot} = 2 \frac{\rho_j V_j^2 - \rho_o V'^2}{\rho_o V_o^2} \frac{s}{c}. \qquad (33)$$

The price of additional lift is jet power (excess-energy flux),

$$(P_j)_{slot} = \Delta p_t \cdot V_j bs. \qquad (34)$$

The jet-power coefficient is defined by

$$(C_{P_j})_{slot} = \frac{(P_j)_{slot}}{q_o V_o bc} = \frac{\Delta p_t}{q_o} \frac{V_j}{V_o} \frac{s}{c}. \qquad (35)$$

For theoretical reasons it may be of interest to state what is left of this power in form of excess-energy flux in the mixing jet at the trailing edge,

$$(C_{P_j})_{te} = \int^{h/c} \frac{\Delta p_t}{q_o} \frac{u}{V_o} d \frac{y}{c}. \qquad (36)$$

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

The experiments to be evaluated in Part B show that, with the present design, it is justified to assume total pressure constant across the blowing slot and static pressure in the blowing slot equal to free-stream static pressure. Under these conditions the jet-momentum coefficient and the stiffness coefficient in the blowing slot, either one divided by the nondimensional slot width, are unique functions of the jet-power coefficient divided by the nondimensional slot width, (Ref. 5). These functions are, with sufficiently small ratios $(s/c)/(C_{pj})_{slot}$:

$$\frac{(C_\mu)_{id}}{s/c} = 2\left(\frac{C_{pj}}{s/c}\right)^{2/3}\left[1+\frac{1}{3}\left(\frac{s/c}{C_{pj}}\right)^{2/3}\right]^{2}, \qquad (37)$$

$$\frac{(C_E)_{id}}{s/c} = 2\left(\frac{C_{pj}}{s/c}\right)^{2/3}\left[1+\frac{1}{3}\left(\frac{s/c}{C_{pj}}\right)^{2/3}\right]^{-1}. \qquad (38)$$

where C_{pj} means $(C_{pj})_{slot}$. The subscript "id", for "ideal", indicates the theoretical values in the blowing slot.

The purpose of the present study is to analyse the empirical relationship between lift and jet power by resolving it into partial problems as illustrated by the following schematic:

$$\begin{tikzcd}
 & (C_{pj})_{slot} \arrow[dl] \arrow[dr] & \\
(C_E)_{slot} \arrow[d] \arrow[rr, dash, "(\delta_f, \alpha)"] & & (C_\mu)_{slot} \arrow[d] \\
(C_E)_{te} & & (C_\mu)_{te}
\end{tikzcd}$$

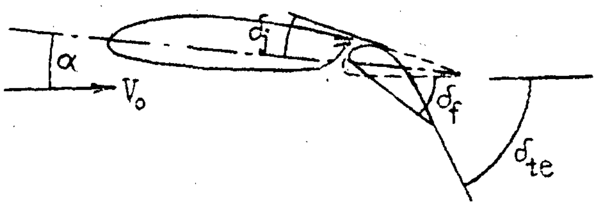
$$C_l = \underbrace{K_1 \sin\alpha + \left(k_1 + k_2\sqrt{(C_E)_{te}}\right)\sin\delta_{te}}_{C_{l\Gamma}} + \underbrace{(C_\mu)_{te}\sin\delta_{te}}_{C_{lM}}. \qquad (39)$$

Ref. 5: H.B. Helmbold, IAS 24 (1957), 339

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

The first partial problem is the relation of stiffness and jet-momentum coefficients in the blowing slot to the jet-power coefficient. This problem has been solved, in good approximation, by Eqs. (37) and (38). The second partial problem is the relation of stiffness and jet-momentum coefficients at the trailing edge to their values in the blowing slot. This relation is controlled by two different influences: the influence of the incoming boundary of the main airfoil which first has to be reenergised before a jet effect can be obtained, and the influence of the pressure distribution along the suction side of the flap on the turbulent mixing process. Both influences depend on flap-deflection angle and angle of attack, as indicated in the schematic. The third problem is the relation of the wing-circulation coefficient to the stiffness coefficient which, in a somewhat crude manner, is empirically solved by the $C_{l\Gamma}$-term of Eq. (39). This equation is supposed to be valid only with fully attached flow. The coefficients K_1, k_1, k_2 are functions of the flap-deflection angle, δ_f. The angle of attack, α, is referred to the zero-lift direction of the basic airfoil, and the jet-deflection angle is $\delta_{te} = \delta_f + \delta_j + \alpha$. (Fig. 2)

Fig.2



As has been seen in Section A2. the stiffness coefficient of the jet sheet increases downstream from the trailing edge toward its asymptotic value, under constant pressure. Actually, the

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

pressure variation along the jet sheet does not impair the former statement. Distinctively from the non-viscous problem where practically C_E = const = $(C_E)_{te}$ along the jet sheet, the jet-induced lift coefficient

$$\Delta C_{l\Gamma} = k_2 \sqrt{(C_E)_{te}} \cdot \sin \delta_{te} \tag{40}$$

is not quite adequately described by a function of the local value of the stiffness coefficient at the trailing edge. It would be better described as a function of an effective value, $(C_E)_{eff}$, of the stiffness coefficient meaning an average value taken over the jet sheet, such that the coefficient k_2 can be taken from non-viscous theory. This average value would be bounded by the inequality

$$(C_E)_{te} < (C_E)_{eff} < 2(C_E)_{te}.$$

Not knowing, however, the effective average, we will use the form (40), keeping in mind that with the physical flow the coefficient k_2 must comprise a hidden factor $\sqrt{(C_E)_{eff}/(C_E)_{te}} > 1$. This factor must be a function of the nondimensional distance from blowing slot to trailing edge, c_f/λ and of secondary parameters like δ_f and α.

4. The Finite-Span Blowing Wing in Viscous Incompressible Flow.

If the two-dimensional lift coefficient, C_l, can be presented adequately by Eq. (39) an approximate solution of the simplest type of the three-dimensional problem (untwisted elliptic wing, elliptic cross section of the jet sheet) is obtained (Ref. 5) from the equation

$$K_1 \sin(\alpha-\alpha_i) + \left[(C_\mu)_{te} + k_1 + k_2\sqrt{(C_E)_{te}}\right] \sin(\delta_{te}-\alpha_i) - (C_\mu)_{te} \times$$
$$\times \sin \alpha_i = \pi A \sin \alpha_i \tag{41}$$

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

where α_i denotes the induced angle of the wing-jet configuration and A is the aspect ratio. The solution is

$$\sin \alpha_i = \frac{M}{\sqrt{M^2 + N^2}} \tag{42}$$

with $M \equiv C_l$ from Eq. (39)

and $N \equiv \pi A + K_1 \cos \alpha + (C_\mu)_{te}(1+\cos \delta_{te}) + (k_1+k_2\sqrt{(C_\mu)_{te}})\cos \delta_{te}$.

This approximate solution is expected to be satisfactory for moderate values of $\sin \alpha_i$, say, smaller than 0.4. The three-dimensional lift coefficient becomes, with approximately constant pressure along the jet sheet,

$$C_L = \left[\pi A + 2\,(C_\mu)_{te}\right] \sin \alpha_i \,.\, \cos \alpha_i. \tag{43}$$

The induced-drag coefficient is

$$C_{Di} = \pi A \sin^2 \alpha_i. \tag{44}$$

Since, with finite aspect ratio, the jet sheet at infinity downstream is still inclined to the free stream velocity a loss of jet thrust results which may be combined with the induced drag,

$$C_{Di} + \Delta C_{Tj} = \left[\pi A + 2(C_\mu)_{te}\right] \sin^2 \alpha_i = C_L \tan \alpha_i. \tag{45}$$

Simplified results can be obtained if $(M/N)^2 \ll 1$. Let us introduce the notation $K_2 \equiv k_1 + k_2\sqrt{C_E}$ and hereafter, in this section, omit the subscripts "te". Then we have, two-dimensionally,

$$M = C_l = K_1 \sin + (C_\mu + K_2) \sin \delta, \tag{46}$$

$$\frac{\partial M}{\partial \alpha} = C_{l\alpha} = K_1 \cos \alpha + (C_\mu + K_2) \cos \delta, \tag{47}$$

$$N = \pi A + C_\mu + K_1 \cos \alpha + (C_\mu + K_2) \cos \delta = \pi A + C_\mu + C_{l\alpha}, \tag{48}$$

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

$$\frac{\partial N}{\partial \alpha} = -K_1 \sin \alpha - (C_\mu + K_2) \sin \delta = -C_1, \tag{49}$$

since $\partial\delta/\partial\alpha = \partial(\delta_f + \delta_j + \alpha)/\partial\alpha = 1$. With $(M/N)^2 \ll 1$ we obtain,

$$\sin \alpha_i = \frac{M}{N} = \frac{C_1}{\pi A + C_\mu + C_{1\alpha}}, \tag{50}$$

$$C_L = (\pi A + 2C_\mu) \sin \alpha_i = \frac{\pi A + 2C_\mu}{\pi A + C_\mu + C_{1\alpha}} C_1,$$

$$C_{L\alpha} = \frac{\pi A + 2C_\mu}{N} \left(\frac{\partial M}{\partial \alpha} - \frac{M}{N}\frac{\partial N}{\partial \alpha}\right)$$

$$= \frac{\pi A + 2C_\mu}{\pi A + C_\mu + C_{1\alpha}} C_{1\alpha} + \left(\frac{C_1}{\pi A + C_\mu + C_{1\alpha}}\right)^2. \tag{51}$$

The second term on the right-hand side of Eq. (51), being identical with $(M/N)^2 \ll 1$, is not negligible in the extreme case when $(\pi A + 2C_\mu)/C_{1\alpha}$ is of the same order of magnitude as M/N or even smaller; therefore, it had to be retained.

The present theory indicates a <u>limitation of the wing-circulation lift</u>. By Eqs. (4) to (6) the wing-circulation lift coefficient is

$$C_{L\Gamma} = C_L - C_\mu \sin \delta. \tag{52}$$

The total-circulation lift coefficient comprises the contribution of the jet sheet and is, according to Eq. (19) of Ref. 5,

$$C_{LC} = \pi A \sin \alpha_i, \cos \alpha_i. \tag{53}$$

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

(In the two-dimensional problem, $A = \infty$, we have $C_{lC} = C_l$ since the momentum lift is completely converted into circulation lift of the jet sheet.) Generally $C_{L\Gamma} < C_{LC}$, except in the special case when $C_\mu = \infty$ because then the jet sheet becomes straight, $c/r = \text{const} = 0$, and cannot support a circulation lift:

$$C_{L\Gamma} = C_{LC} \quad \text{for } C_\mu = \infty . \tag{54}$$

For a given aspect ratio

$$C_{LC} = \pi A \frac{MN}{M^2 + N^2} \tag{55}$$

has its maximum

$$C_{LC} = \max = \frac{\pi}{2} A \tag{56a}$$

$$\text{for} \quad \frac{M}{N} = \frac{C_\mu \sin\delta + K_1 \sin\alpha + K_2 \sin\delta}{\pi A + C_\mu(1+\cos\delta) + K_1 \cos\alpha + K_2 \cos\delta} = 1 . \tag{56b}$$

For $C_\mu = \infty$, $C_{L\Gamma} = C_{LC}$ has the same maximum

$$C_{L\Gamma} = \max = \frac{\pi}{2} A \text{ for } \delta = \frac{\pi}{2} \tag{57}$$

since, with $C_\mu = \infty$, the condition (56b) is satisfied by $\delta = \pi/2$. Experimentally, this limiting value is never reached inasmuch as, according to Fig. 4 of Ref. 6 where $\delta = \pi/2$, $C_{L\Gamma}$ for a given aspect ratio has its maximum at a finite value of C_μ. The slope $d(C_{L\Gamma})_{max.}/dA$ of the envelope is about half the theoretical value at $A = 0$ and approaches the theoretical value, $\pi/2$, with increasing aspect ratio. The theory does not account

Ref. 6: J.G. Lowry - R.D. Vogler, NACA-TN3863 (1956)

A. FUNDAMENTALS OF THE THEORY OF THE BLOWING WING (Cont'd)

for the decrease of jet sheet curvature with decreasing aspect ratio, the rolling-in deformation of the jet sheet and any leading-edge separation. At the present we cannot assess with certainty the relative importance of these phenomena. Presumably, leading-edge separation is the factor principally responsible for the lift deficiency, at least at high aspect ratios. At $A = 10$ the experimental value of $(C_{L\Gamma})_{max}$ is about 12.84 instead of the theoretical value 15.71.

B. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND WIDE BLOWING SLOT

Wind tunnel tests on a blowing NACA 23015 airfoil with a 25% chord slotted flap were carried out at the University of Wichita (Ref. 7). The width of the blowing slot was adjustable. This Part B of the present report deals with the experiments with a blowing-slot width of 0.9% airfoil chord. In these experiments the total-pressure ratio in the blowing slot, $p_{tj}/p_{to} \leq 1.152$, was sufficiently low to permit the mixing flow at the trailing edge to be treated as incompressible. The analysis has been restricted to conditions where the flow was attached to the flap. The principal data of the analysis are collected in Tables 1a and 1b. Angles of attack are referred either to the airfoil chord (α_c) or to the zero lift axis of the basic airfoil ($\alpha = \alpha_c + 1°$). The upper tangent at the trailing edge is inclined to the zero lift axis by $\delta_j = 10°$. Hence, the jet-deflection angle is $\delta_{te} = \delta_f + \alpha + 10° = \delta_f + \alpha_c + 11°$.

B. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND WIDE BLOWING SLOT

TABLE 1a.

δ_f°	α_c°	C_{pj} slot	$(C_{pj})_{te}$	$(C_\mu)_{id}$	C_μ slot	$(C_\mu)_{te}$	$(C_E)_{id}$	C_E slot	$(C_E)_{te}$
30	+0.1	0.0544	0.0142	0.0702	0.0662	0.0430	0.0544	0.0516	0.0236
	.1	.9745	.3410	.421	.4202	.3008	.402	.4110	.3044
	-0.1	4.7075	1.5884	1.182	1.1872	.8559	1.165	1.1874	.8744
	5.2	0.0525	0.0079	0.0710	0.0662	0.0370	0.0529	0.0506	0.0136
	5.2	.9861	.3479	.424	.4202	.2951	.407	.4094	.274?
	5.1	4.697	1.5790	1.178	1.1872	.8324	1.160	1.1784	.8478
	10.3	0.9698	0.3314	0.3915	0.4202	0.2886	0.3735	0.4107	0.2855
	10.2	4.697	1.4916	1.179	1.1872	.7985	1.161	1.1803	.8200
60	+0.1	0.3687	0.1798	0.226	0.2089	0.1887	0.208	0.2114	0.2022
	.1	1.5171	.8020	.561	.5677	.5624	.543	.5508	.5832
	-0.05	4.555	2.296	1.155	1.1574	1.1101	1.138	1.1458	1.2306
	5.3	0.3681	0.1729	0.2255	0.2089	0.1843	0.2075	0.2092	0.2034
	5.2	1.5204	.8299	.561	.5677	.5630	.544	.5464	.5864
	5.1	4.432	2.653?	1.132	1.1574	1.166?	1.116	1.1276	1.311?
	10.4	0.3823	0.1521	0.2312	0.2089	0.1710	0.2133	0.2172	0.1888
	9.3	1.5158	.7370	.561	.5677	.4936	.543	.5509	.5539
70	+0.2	0.7022	0.3560	0.3405	0.3220	0.3039	0.323	0.3136	0.3436
	.1	1.4362	.8240	.541	.5744	.5166	.524	.5534	.6020
	0.1	4.324	2.162	1.114	1.1270	.9974	1.097	1.1188	1.1878
	5.3	0.6554	0.3396	0.3260	0.3220	0.2952	0.308	0.3090	0.3298
	5.2	1.4927	.7762	.555	.5744	.5296	.537	.5460	.569?
	10.4	0.6559	0.3003	0.3262	0.3220	0.2738	0.3088	0.3089	0.3111
	10.3	1.4725	.8238	.550	.5744	.5477	.533	.5431	.6154

B. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND WIDE BLOWING SLOT

Table 1b.

δ_f^o	α_c^o	δ_{te}^o	$C_{pj}^{1/3}$ slot	C_l	$(C_\mu \sin \delta)_{te}$	$C_{l\Gamma}$	$(\sqrt{C_E}\sin \delta)_{te}$	C_d	C_m
30	+0.1	41.1	0.380	2.140	0.028	2.112	0.101	-0.0227	-0.417
	.1	41.1	.9925	3.263	.198	3.065	.363	-0.3068	-0.738
	-0.1	40.9	1.677	4.551	.560	3.991	.612	-0.9143	-1.211
	5.2	46.2	0.3746	2.662	0.027	2.635	0.0842	-0.0033	-0.408
	5.2	46.2	.986	3.930	.213	3.717	.3785	-0.2723	-0.748
	5.1	46.1	1.685	5.228	.600	4.628	.664	-0.8135	-1.197
	10.3	51.3	0.990	4.484	0.225	4.259	0.417	-0.2215	-0.733
	10.2	51.2	1.675	5.927	.622	5.305	.706	-0.7224	-1.204
60	+0.1	71.1	0.718	-----	0.179	-----	0.426	+0.0258	-0.985
	.1	71.1	1.148	5.769	.532	5.237	.726	-0.1428	-1.340
	-0.05	71.0	1.658	7.487	1.050	6.437	1.048	-0.4032	-1.929
	5.3	76.3	0.7167	-----	0.179	-----	0.4385	+0.0679	-0.958
	5.2	76.2	1.141	6.167	.546	5.621	.7445	-0.0765	-1.298
	5.1	76.1	1.643	7.929	1.132	6.797	1.112	-0.2509	-1.875
	10.4	81.4	0.726	-----	0.169	-----	0.4295	+0.1123	-0.865
	9.3	80.3	1.145	6.606	.487	6.119	.734	-0.03067	-1.278
70	+0.2	81.2	0.888	5.343	0.300	5.043	0.579	+0.084	-1.135
	.1	81.1	1.122	6.205	.510	5.695	.767	.0092	-1.418
	-0.1	80.9	1.629	8.030	.985	7.045	1.075	-0.1133	-2.042
	5.3	86.3	0.869	5.627	0.295	5.332	0.5735	+0.132	-1.072
	5.2	86.2	1.125	6.722	.529	6.193	.7535	.0813	-1.400
	10.4	91.4	0.869	5.663	0.274	5.389	0.5575	+0.163	-0.955
	10.3	91.3	1.138	6.702	.547	6.155	.785	..1165	-1.319

B. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND WIDE BLOWING SLOT

1. Comparison of Theoretical and Experimental Lift.

According to Ref. 8 the wing-circulation lift coefficient for a non-mixing jet and for small angles is

$$C_{l\Gamma} = C_l - (C_\mu)_{te}\,\delta_{te} = F_1\,\alpha + F_\tau \delta_f + F_o \delta_j \qquad (58)$$

where F_1 and F_o are functions of $(C_E)_{te}$ only and F_τ is a function of $(C_E)_{te}$ and the flap-chord ratio c_f/c. For larger angles and finite thickness ratio, $t/c = 0.15$, Eq. (28) is tentatively modified to the formula

$$C_{l\Gamma} = C_l - (C_\mu)_{te} \sin \delta_{te} = (1 + 0.77\,\frac{t}{c})F_1 \sin (\alpha + \frac{F_\tau}{F_1}\,\delta_f + \frac{F_o}{F_1}\,\delta_j) \qquad (59)$$

The coefficients F_1, F_τ/F_1 and F_o/F_1 are presented in Table 2 as functions of $\sqrt{(C_E)_{te}}$, for a flap-chord ratio $c_f/c = 0.25$.

TABLE 2.

$\sqrt{(C_E)_{te}}$	F_1	F_τ/F_1	F_o/F_1
0.0	6.283	0.609	0.000
.2	6.518	.615	.105
.4	6.764	.634	.195
.6	7.024	.657	.273
.8	7.299	.682	.340
1.0	7.592	.705	.450
1.2	7.89	.728	.450

$c_f/c = 0.25$

Ref. 7: P.E. Morris, University of Wichita, Engineering Rep. 273 (April 1957) = FAD Report R246-005

Ref. 8: D.A. Spence, RAD-TN Aero2450 (May 1956)

B. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND WIDE BLOWING SLOT

Theoretical values of $C_{l\Gamma}$ from Eq. (59) have been computed for $\alpha = 1.1^\circ$, $\delta_f = 30^\circ, 60^\circ, 70^\circ$ and $\delta_j = 10^\circ$ and are compared with the experimental values in Fig. 3. By extrapolating the experimental curves to $(C_E)_{te} = 0$ it is seen that there is a loss of lift which must be caused by leading edge separation since the curves are based on measurements with flow attached to the flap. The loss increases with the flap deflection as shown by Table 3.

TABLE 3.

δ_f°	$C_{l\Gamma}$ at $(C_E)_{te} = 0$		Loss %
	Eq. (59)	Experiment	
30	2.325	1.737	25.3
60	4.29	2.500	41.8
70	4.84	2.668	44.9

It is not certain whether the flow was truly two-dimensional in these tests because the airfoil was not shielded by fences from the wind tunnel wall boundary layer. Therefore, it is possible that the loss as given by Table 3 is greater than it would be in truly two-dimensional flow. The slopes of the experimental curves are much steeper than the theoretical slopes as must be expected after the discussions of Sections A2 (stiffening effect) and A3 (sink effect). This shows that, from the practical standpoint, present-day theory is seriously deficient because it ignores the effects of jet mixing. Therefore, at the present, the designer must rely on experiments guided and interpreted by a qualitative theory covering all of the pertinent physical phenomena.

B. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND WIDE BLOWING SLOT

2. An Empirical Interpolation Formula for Wing-Circulation Lift.

The experimental values of $C_{l\Gamma}$ with flow attached to the flap can be presented by the formula

$$C_{l\Gamma} = c_1 \sin\alpha + k_1 \sin\delta_o + k_2 \sqrt{(C_E)_{te}} \sin\delta_{te} \tag{60}$$

where $\delta_o \equiv \delta_p + \delta_j$ and the coefficients c_1, k_1, k_2 are functions of δ_o. Since

$$\sin\delta_{te} = \sin(\delta_o + \alpha) = \sin\delta_o \cos\alpha + \cos\delta_o \sin\alpha,$$

$$\sin\delta_o = (\sin\delta_{te} - \cos\delta_o \sin\alpha)/\cos\alpha$$

Eq. (60) becomes

$$C_{l\Gamma} = \left(c_1 - k_1 \frac{\cos\delta_o}{\cos\alpha}\right)\sin\alpha + \left(\frac{k_1}{\cos\alpha} + k_2\sqrt{(C_E)_{te}}\right)\sin\delta_{te} \tag{61}$$

For small angles of attack this simplifies to

$$C_{l\Gamma} = (c_1 - k_1\cos\delta_o)\sin\alpha + \left(\frac{k_1}{\cos\alpha} + k_2\sqrt{(C_E)_{te}}\right)\sin\delta_{te} \tag{62}$$

$$\equiv K_1 \sin\alpha + K_2 \sin\delta_{te}$$

where $K_1 \equiv c_1 - k_1 \cos\delta_o$ and $K_2 \equiv k_1 + k_2\sqrt{(C_E)_{te}}$. The coefficients of Eqs. (60) and (62) are presented by Table 4.

B. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND WIDE BLOWING SLOT

Table 4.

δ_f^o	δ_o^o	c_1	k_1	k_2	K_1
30	40	5.875	2.527	3.683	3.94
45	55	5.67	2.53	3.71	4.22 (interpolated)
60	70	5.36	2.550	3.794	4.49
70	80	5.00	2.610	4.108	4.54

By adding the momentum-lift coefficient $(C_\mu)_{te} \cdot \sin \delta_{te}$ to Eq. (62) we obtain for the total lift coefficient the expression (39) on which the three-dimensional theory (Section A4) is based.

3. Connection between Jet Coefficients at Trailing Edge and Blowing Slot.

In this section we are dealing with the second partial problem mentioned toward the end of Section A3, right after Eq. (39). One of the influences controlling the connection between the jet coefficients at trailing edge and blowing slot is the influence of the pressure distribution along the suction side of the flap on the mixing jet. This is illustrated by Figs. 4 and 5 demonstrating the situation at relatively high jet coefficients where the other influence, the influence of the incoming boundary layer of the main airfoil, has become subordinate. Fig. 4 shows that stiffness coefficient, momentum coefficient and excess-energy coefficient at the trailing edge have almost simultaneously their maximum at a flap deflection angle near 60°. The influence of the static pressure distribution is qualitatively the same on all of these coefficients. The stiffening effect on the

B. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND WIDE BLOWING SLOT

stiffness coefficient explains the fact that $(C_E)_{te}$ exceeds $(C_E)_{slot}$ at the high deflection angles. At the end of Section A2 an excess of 15.4% was predicted for the special case of constant static pressure and no losses by skin friction. The static pressure distributions are presented by Fig. 5. There are two suction peaks at the higher flap deflection angles; the first one is the regular one occurring in potential flow, the second one is the result of mutual attraction between the convex flap contour and the fast mixing jet attaching to it. At the low flap deflection angle both suction peaks melt together. It is conjectured that without skin friction the 70°-distribution would be best because it diminishes most the relative velocities between mixing jet and potential flow, and that this advantage is overcome by the increase of skin friction from 60° to 70°. (The second suction peak decreases and finally vanishes with decreasing jet-momentum coefficients for any higher flap deflection because with low jet-momentum coefficients the mixing jet never attaches to the deflected flap and the flow separates therefrom. These states of flow have been excluded from the analysis.)

Since, with the present configuration, the static pressures in the blowing slot are not too different from free-stream static pressure it is preferable to replace $(C_\mu)_{slot}$ and $(C_E)_{slot}$ by $(C_\mu)_{id}$ and $(C_E)_{id}$, respectively, the latter values being related to $(C_{pj})_{slot}$ by Eqs. (37) and (38). This replacement is justified by the figures of Table 1a.

The experimental curves $(C_\mu)_{te}$ vs. $(C_\mu)_{id}$, Fig. 6, and $(C_E)_{te}$ vs. $(C_E)_{id}$, Fig. 7, can be represented by functions of the type

$$y = \frac{Ax + Bx^2 - C}{1 + Ax} \tag{63}$$

B. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND WIDE BLOWING SLOT

where y denotes the trailing-edge values and x the corresponding ideal values. The derivatives with respect to x are $y'(0) = A(1 + C)$ and $y'(\infty) = B/A$. Their ratio $y'(\infty)/y'(0) = B/[A^2(1 + C)] < 1$ indicates a negative overall curvature of the curves. Evaluation of the coefficients from experimental values suggests the hypotheses that B/A^2 is a linear function of $\cos \delta_f$ and C is a linear function of δ_f^2 for both the C_μ-relation and the C_E-relation. Under these conditions it happens that $y'(\infty)/y'(0)$ becomes practically the same function of δ_f for both relations. Since this appears to be a reasonable correlation smoothed values of the coefficients A, B, C for the C_μ-relation and A', B', C' for the C_E-relation have been determined according to the above hypotheses and are presented in Table 5.

TABLE 5.

Coefficients for $(C_\mu)_{te}$ vs. $(C_\mu)_{id}$

δ_f^o	A	B	C	$y'(\infty)/y'(0)$
30	0.760	0.4725	0.0021	0.816
45	1.131	.863	.0585	.637 (interpolated)
60	1.481	1.069	.1375	.428
70	1.673	0.964	.2027	.2826

Coefficients for $(C_E)_{te}$ vs. $(C_E)_{id}$

δ_f^o	A'	B'	C'	$y'(\infty)/y'(0)$
30	0.8772	0.570	0.0220	0.815
45	1.242	1.066	.0886	.636 (interpolated)
60	1.646	1.373	.182	.429
70	2.071	1.567	.259	.2905

B. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND WIDE BLOWING SLOT

The curves in Figs. 6 and 7 have been computed from these smoothed values. No systematic influence of the angle of attack was discernible except at $\delta_f = 30^\circ$. Similarly to Fig. 3, the intersection of the extrapolated curves with the abscissa axis indicates leading-edge separation at the higher flap-deflection angles. There is however the difference that at $\delta_f = 30^\circ$ there is definitely no such separation. Since pressure surveys underlying Figs. 6 and 7 are carried out at mid-span station this difference would be explained if the assumption is true that premature separation is caused by the presence of the wind tunnel wall boundary layer which means that the flow is not truly two-dimensional. At $\delta_f = 30^\circ$ the separation would be confined to the neighborhood of the wall.

4. Extrapolation of the Lift-Power Relationship to Higher Power Coefficients.

The lift-power relationship is established by the compound of Eq. (39) with the coefficients of Table 4, Eq. (63) with the coefficients of Table 5, and Eqs. (37) and (38). The result of extending these computations over a much wider range of jet-power coefficients than in the wind tunnel tests is shown in Fig. 8. The full lines indicate flow attached to the flap, the broken lines indicate flow separated from and re-attaching to, the flap.

5. Drag of the Blowing Airfoil.

For a given set of flap-deflection angle and angle of attack the total-drag coefficient is empirically a linear function of the trailing-edge momentum coefficient

$$c_d = c_{do} + c_d' \cdot (c_\mu)_{te} . \qquad (64)$$

B. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND WIDE BLOWING SLOT

Except for one point, the values of the slope, C_d', plotted against the zero value, C_{do}, arrange themselves along a single curve, Fig. 9. The sequence of the parameters δ_f and α_c along the curve indicates a continuously increasing loss of leading-edge thrust (increasing leading-edge drag) with increasing angles. This is consistent with the inferences from Figs. 3,6 and 7. Without leading-edge separation, C_d' should be essentially independent of C_{do} and not much different from -1.

C. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND NARROW BLOWING SLOT

This Part C of the present report deals with the wind tunnel tests on the same model as in Part B, with the only difference that the blowing-slot width was decreased from 0.9% to 0.15% airfoil chord. Because of the higher total-pressure ratio in the blowing slot, $p_{tj}/p_{to} \leq 1.693$, compressibility effects were not always negligible. The basic data of the analysis are collected in Tables 6a and 6b.

TABLE 6a

δ_f^o	α_c^o	C_{P_j} slot	$(C_\mu)_{id}$	$(C_\mu)_{te}$	$(C_E)_{id}$	$(C_E)_{te}$
33	0.1	0.2671	0.0969	0.0609	0.0940	0.0564
30	.1	1.0550	.2395	.1922	.2360	.1819
	.0	6.013	.760	.5320	.757	.5755
33	5.2	0.2669	0.0969	0.0418	0.0940	0.0328
30	5.2	1.0545	.2395	.1672	.2360	.1791
	5.1	6.013	.760	.5304	.757	.6097
30	10.3	1.0540	0.2395	0.1578	0.2360	0.1837
	10.2	6.014	.760	.5546	.757	.6374
60	0.1	0.7103	0.1840	0.1540	0.1813	0.1823
	.1	1.8480	.347	.2910	.3436	.3384
	.0	5.471	.712	.5547	.710	.6632
	5.2	0.7102	0.1840	0.1481	0.1813	0.1755
	5.2	1.8479	.347	.2841	.3436	.3259
	5.1	5.469	.712	.5494	.710	.6581
	10.4	0.7102	0.1840	0.1389	0.1813	0.1652
	9.3	1.8477	.347	.2611	.3436	.3128
70	0.2	0.3164	0.1083	0.0900	0.1053	0.0861
	.2	.6781	.1784	.1551	.1757	.1806
	.1	1.7874	.339	.3114	.336	.3545
	.0	5.235	.692	.5973	.690	.7110
	5.3	0.3164	0.1083	0.0822	0.1053	0.0748
	5.3	.6777	.1784	.1573	.1757	.1770
	5.2	1.7872	.339	.2989	.336	.3536
	5.1	5.234	.692	.6487	.690	.7781
	10.4	0.6776	0.1784	0.1479	0.1757	0.1680
	7.3	1.7869	.339	.3056	.336	.3680

TABLE 6b

δ_f^o	δ_{te}^o	C_l	$(C_\mu \sin\delta)_{te}$	C_l	$(\sqrt{C_E}\sin\delta)_{te}$	$C_{P_j}^{1/3}$	C_d	C_m
33	44.1	2.500	0.0424	2.458	0.1653	0.644	-0.0455	-0.505
30	41.1	3.108	.1264	2.974	.2808	1.018	-	-0.689
	41.0	4.180	.349	3.731	.4975	1.319	-	-1.088
33	49.2	3.057	0.0316	3.205	0.1370	0.644	-0.0287	-0.505
30	46.2	3.704	.1206	3.583	.3054	1.018	-	-0.688
	46.1	4.879	.382	4.497	.558	1.819	-	-1.097
30	51.3	4.312	0.1231	4.139	0.335	1.018	-	-0.689
	51.2	5.515	.424	5.091	.6225	1.819	-	-1.090
60	71.1	4.931	0.1456	4.785	0.404	0.892	+0.0061	-1.083
	71.1	5.75	.281	5.47	.550	1.226	-0.0863	-1.354
	71.0	6.736	.525	6.211	.770	1.762	+0.057?	-1.722
	76.2	5.526	0.1438	5.382	0.407	0.892	+0.0408	-1.085
	76.2	6.273	.276	5.997	.554	1.226	-0.0380	-1.344
	76.1	7.452	.534	6.918	.788	1.762	+0.049?	-1.749
	81.4	6.002	0.1372	5.865	0.402	0.892	+0.0721	-1.042
	80.3	6.554*	.2573	6.297	.551	1.226	.0106	-1.292
70	81.2	4.632	0.0889	4.543	0.290	0.682	0.0720	-0.955
	81.2	5.340	.1532	5.187	.420	.878	.0798	-1.149
	81.1	6.263	.308	5.955	.589	1.214	.0307	-1.438
	81.0	7.602	.590	7.012	.834	1.736	-	-1.915
	86.3	4.981	0.0820	4.899	0.273	0.682	0.0937	-0.898
	86.3	5.831	.1569	5.674	.420	.878	.1167	-1.135
	86.2	6.694	.288	6.406	.594	1.214	.0859	-1.401
	86.1	7.952*	.647	7.305	.881	1.736	-	-1.867
	91.4	6.148*	0.1479	6.000	0.410	0.878	0.1476	-1.063
	88.3	6.911*	.3055	6.605	.607	1.214	.1141	-1.393

*near stall

C. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND NARROW BLOWING SLOT

1. Modification of Basic Formulae for Compressibility Effects.

The expression for the jet-power coefficient, Eq. (35), has to be replaced by

$$(c_{P_j})_{slot} = \frac{(P_j)_{slot}}{q_o V_o bc} = \frac{g\, c_p\, T_{to} \cdot m_j}{q_o V_o bc} \left[\left(\frac{p_{tj}}{p_{to}} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right] \tag{65}$$

where $m_j = \rho_j V_j bs$.

Theoretically, the ideal jet-momentum and stiffness coefficients should be modified by multiplying the right-hand sides of Eqs. (37) and (38) by the factor $(\rho_s/\rho_j)^{1/3}$ where

$$\frac{\rho_S}{\rho_j} = \frac{T_{tj}}{T_{to}} \left(\frac{p_{to}}{p_{tj}} \right)^{\frac{\kappa-1}{\kappa}} \tag{66}$$

since, with a subsonic jet, $p_s = p_j$. This correction, however, was neglected because it was smaller than the experimental scatter.

The total-momentum coefficient at the trailing edge is defined by Eq. (30). The velocities and densities occurring in that equation are evaluated by the formulae

$$u = \sqrt{2\, gc_p T_t \left[1 - \left(\frac{p}{p_t} \right)^{\frac{\kappa-1}{\kappa}} \right]}, \tag{67}$$

$$V' = \sqrt{2\, gc_p T_{to} \left[1 - \left(\frac{p}{p_{to}} \right)^{\frac{\kappa-1}{\kappa}} \right]}, \tag{68}$$

C. ANALYSIS OF EXPERIMENTS ON A TWO DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND NARROW BLOWING SLOT

$$V_j = \sqrt{2g\, c_p T_{tj} \left[1 - \left(\frac{p_j}{p_{tj}}\right)^{\frac{\kappa-1}{\kappa}}\right]}, \tag{69}$$

$$\rho_j V_j = \frac{m_j}{b\ell}, \tag{70}$$

$$\frac{\rho_j}{\rho_o} = 1/(1 + \frac{u - V'}{V_j - V_o} \quad \frac{T_{tj} - T_{to}}{T_{to}}). \tag{71}$$

The stiffness coefficient at the trailing edge is defined by Eq. (31) where

$$\rho u^2 - \rho_o V'^2 = \frac{2\kappa}{\kappa-1} \left[\left(\frac{p_t}{p}\right)^{\frac{\kappa-1}{\kappa}} - \left(\frac{p_{to}}{p}\right)^{\frac{\kappa-1}{\kappa}} \right] \tag{72}$$

2. Comparison of Theoretical and Experiment Lift.

According to Eq. (59) the theoretical values of C_l are independent of the blowing-slot width ratio. Experimental values of C_l at $\alpha = 1.1^o$ extrapolated to $(C_E)_{te} = 0$ are presented in Table 7.

Table 7.

δ_f^o	$C_{l\Gamma}$ at $(C_E)_{te} = 0$		Loss %
	Eq.(59)	Experiment	
30	2.325	1.987	14.5
60	4.29	3.230	24.1
70	4.84	3.312	31.6

C. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND NARROW BLOWING SLOT

Comparison of Tables 3 (wide slot) and 7 (narrow slot) shows that the loss of wing-circulation lift is considerably decreased by the change from the wide to the narrow blowing slot. This is explained by the considerations that, with equal jet momentum, the sink effect is stronger for the narrow slot than for the wide slot, and that a stronger sink effect results in an eariler reattachment of the flow separated at the trailing edge and, consequently, in a smaller loss of lift. The conclusion about the sink effect can be checked from the influence of blowing-slot width on the static pressure on the suction side near the blowing slot. With a non-mixing jet the pressure coefficient C_p at a given station on the airfoil contour, for fixed values of α and δ_f, is a linear function of the wing-circulation lift coefficient $C_{l\Gamma}$ only, independent of blowing-slot width. Under these conditions $C_{l\Gamma}$ can be varied only by variation of $(C_E)_{te}$ or $(C_\mu)_{slot}$. With the mixing jet the pressure coefficient is influenced by its sink effect which, in turn, is a function of $(C_\mu)_{slot}$. Differences in the sink effect must change the relationship between C_p at a given point and $C_{l\Gamma}$, for fixed values of α and δ_f. This relationship is shown in Fig. 10 for both blowing-slot width ratios under consideration, at fixed values of $\alpha = 1.1^\circ$ and δ_f. The station of the static pressure tap, No. 16, was at 76.26% of airfoil chord on the suction side. The slopes of the curves $-C_{p16}$ vs $C_{l\Gamma}$ are presented in Table 8.

Table 8.

$\frac{d(-C_{p16})}{d\,C_{l\Gamma}}$	$\frac{s}{c} = 0.0015$	$\frac{s}{c} = 0.0090$	Difference
$\delta_f = 30^\circ$	1.308	0.815	0.493
60°	1.410	1.042	.368
70°	1.39?	1.194	.20?

C. ANALYSIS OF EXPERIMENTS ON A TWO DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND NARROW BLOWING SLOT

The slopes are systematically steeper for the narrow slot than for the wide slot, a fact to be explained only by a stronger sink effect.

3. Coefficients for the Computation of Wing-Circulation Lift.

The formulae for $C_{L\Gamma}$, Eqs. (60) and (62), are valid for both blowing-slot width ratios, with different coefficients. The coefficients for the narrow slot are presented in Table 9.

Table 9.

δ_f	δ_o^o	c_1	k_1	k_2	K_2
30	40	5.175	2.934	3.455	2.925
45	55	5.09	3.24	3.70	2.97 (interpolated)
60	70	4.96	3.333	4.02	3.84
70	80	4.81	3.270	4.35	4.24

According to Eq. (40), k_2 is the coefficient of jet-induced lift. Comparison of Table 9 with Table 4 shows that this coefficient is not much affected by the change from the wide to the narrow blowing slot, the difference being within the margin of + 7% over the investigated range of flap deflections. The k_1-values reflect the sink effect on leading-edge separation discussed in the preceding Section C2. No explanation has been found for the decrease of the c_1-values.

C. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND NARROW BLOWING SLOT

4. Connection between Jet Coefficients at Trailing Edge and Blowing Slot.

The experimental curves $(C_\mu)_{te}$ vs. $(C_\mu)_{id}$ and $(C_E)_{te}$ vs. $(C_E)_{id}$ are plotted in Fig. 11. Their comparison with Figs. 6 and 7 confirms the conclusion of Section C2. that the leading-edge separation is less developed than with the wide blowing slot, as can be seen from the intersection of the curves with the abscissa axes. The curves for $\delta_f = 30^\circ$ remain practically unchanged by the variation of blowing-slot width. At higher abscissa values the ordinates increase monotonically with increasing flap deflection; there is no optimum deflection angle in the investigated range. At the higher flap-deflection angles, $\delta_f = 60^\circ$ and 70°, the ordinates are higher at low abscissa values and lower at high abscissa values than with the wide blowing slot. The latter phenomenon is the effect of a change in the static pressure distributions for the higher flap deflections, Fig. 12. With the narrow blowing-slot width the first suction peaks are less developed than with the wide slot because the main airfoil extends to 80.93% airfoil chord, with the narrow slot, instead of 78.44% airfoil chord, with the wide slot.

The experimental curves can be represented by functions of the type (63). Smoothed values of the coefficients are presented in Table 10.

C. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND NARROW BLOWING SLOT

Table 10.
Coefficients for $(C_\mu)_{te}$ vs $(C_\mu)_{id}$

δ_f	A	B	C	$y'(\infty)/y'(0)$	
30	0.760	0.4725	0.0021	0.816	
45	.868	.520	.0163	.668	(interpolated)
60	1.052	.5805	.0374	.505	
70	1.256	.627	.0543	.377	

Coefficients for $(C_E)_{te}$ vs. $(C_E)_{id}$

	A′	B′	C′	$y'(\infty)/y'(0)$	
30	0.8772	0.570	0.0220	0.815	
45	1.018	.730	.0370	.679	(interpolated)
60	1.284	.883	.0580	.567	
70	1.564	.998	.0754	.380	

5. The Effect of Blowing-Slot Width on the Lift-Power Relationship.

The total-lift coefficients have been plotted against the cubic root of the jet-power coefficient for both blowing-slot width ratios in Fig. 13. The diagram contains a set of lift curves at $\alpha = 1.1^\circ$ and $\delta_f = 30^\circ, 60^\circ, 70^\circ$ for each width ratio, and, furthermore, a set of lift curves at $\delta_f = 70^\circ$ and variable α for the wide slot only.

The discussion of the slot-width effect is simplest for $\delta_f = 30^\circ$ where there is no appreciable effect of slot width on the relation between jet coefficients at trailing edge and blowing slot. Independently of flap deflection angle the jet-power requirement for given ideal values

C. ANALYSIS OF EXPERIMENTS ON A TWO-DIMENSIONAL AIRFOIL WITH SLOTTED FLAP AND NARROW BLOWING SLOT

of jet-momentum and stiffness coefficients is higher for the narrow slot. According to Eqs; (37) and (38) the jet-power coefficient $(C_{P_j})_{slot}$ is approximately proportional to $(s/c)^{-1/2}$. In our case, with the wide slot 6.0 times wider than the narrow one, the narrow slot would require a C_{P_j}-value about 2.45 times higher than required by the wide slot if the lift coefficient were determined by the ideal jet coefficients only; the values of $(C_{P_j})^{1/3}$ at equal C_l would differ by a factor about 1.38. Actually, the direct effect just defined is counteracted by the sink effect on leading-edge separation discussed in Section C2: the lift coefficients for the narrow blowing slot are somewhat higher than predicted by the first consideration.

With the higher flap deflections, $\delta_f=60^{\circ}$ and 70°, the sink effect on leading-edge separation is strong enough to overcome the direct effect at the lower values of C_{P_j}. With increasing C_{P_j}, however, the direct effect becomes prevalent and is supported by the adverse effect of narrow slot width on the relation between jet coefficients at trailing edge and blowing slot.

With the narrow blowing slot the limitation of the lift by stall is clearly recognizable for the highest flap-deflection angle, $\delta_f = 70^{\circ}$. The experimental data for the wide blowing slot are not quite sufficient to determine the stalling condition; there is, however, no doubt that the stall curve is considerably steeper with the side slot.

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Fig.1

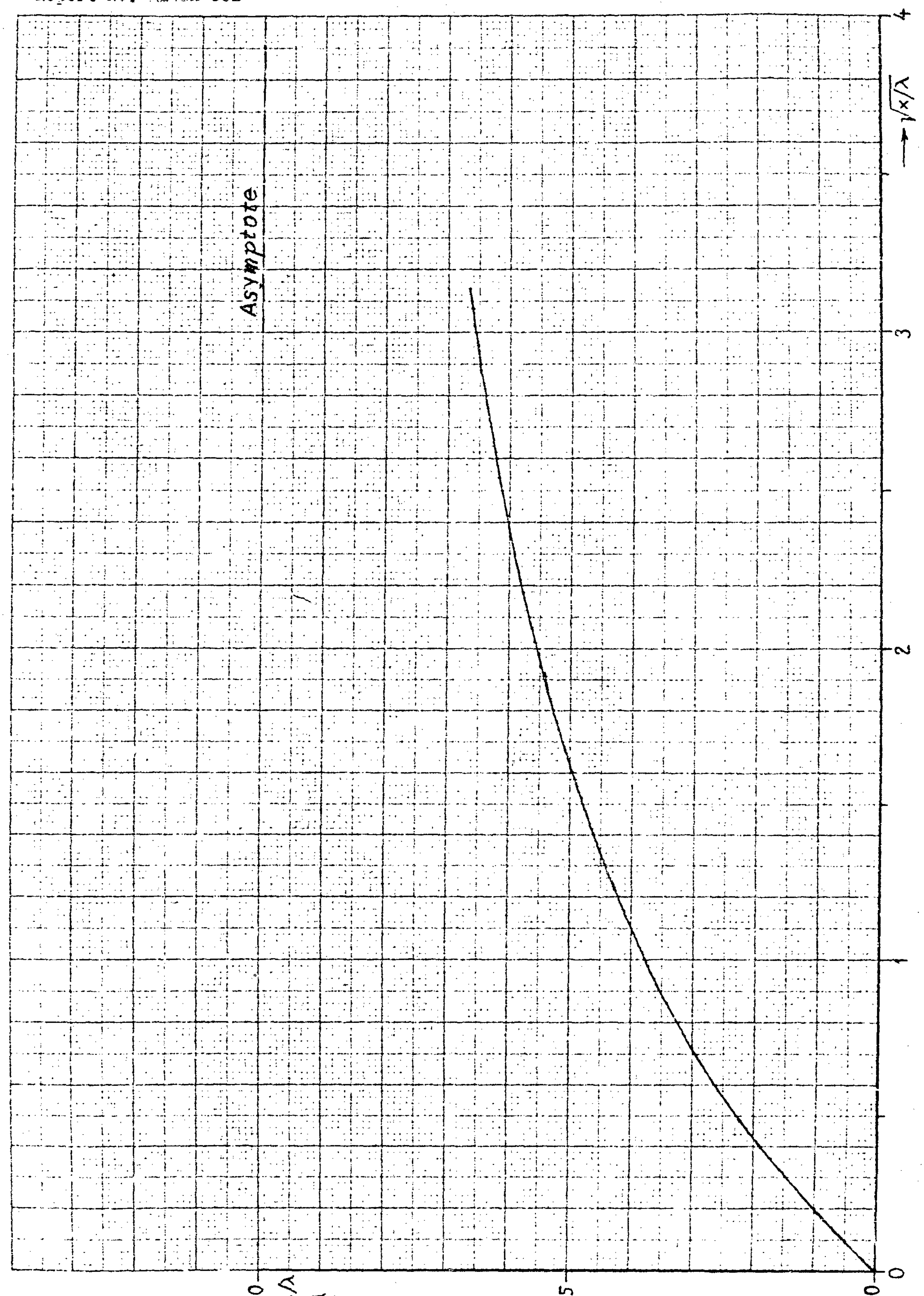


Fig.3

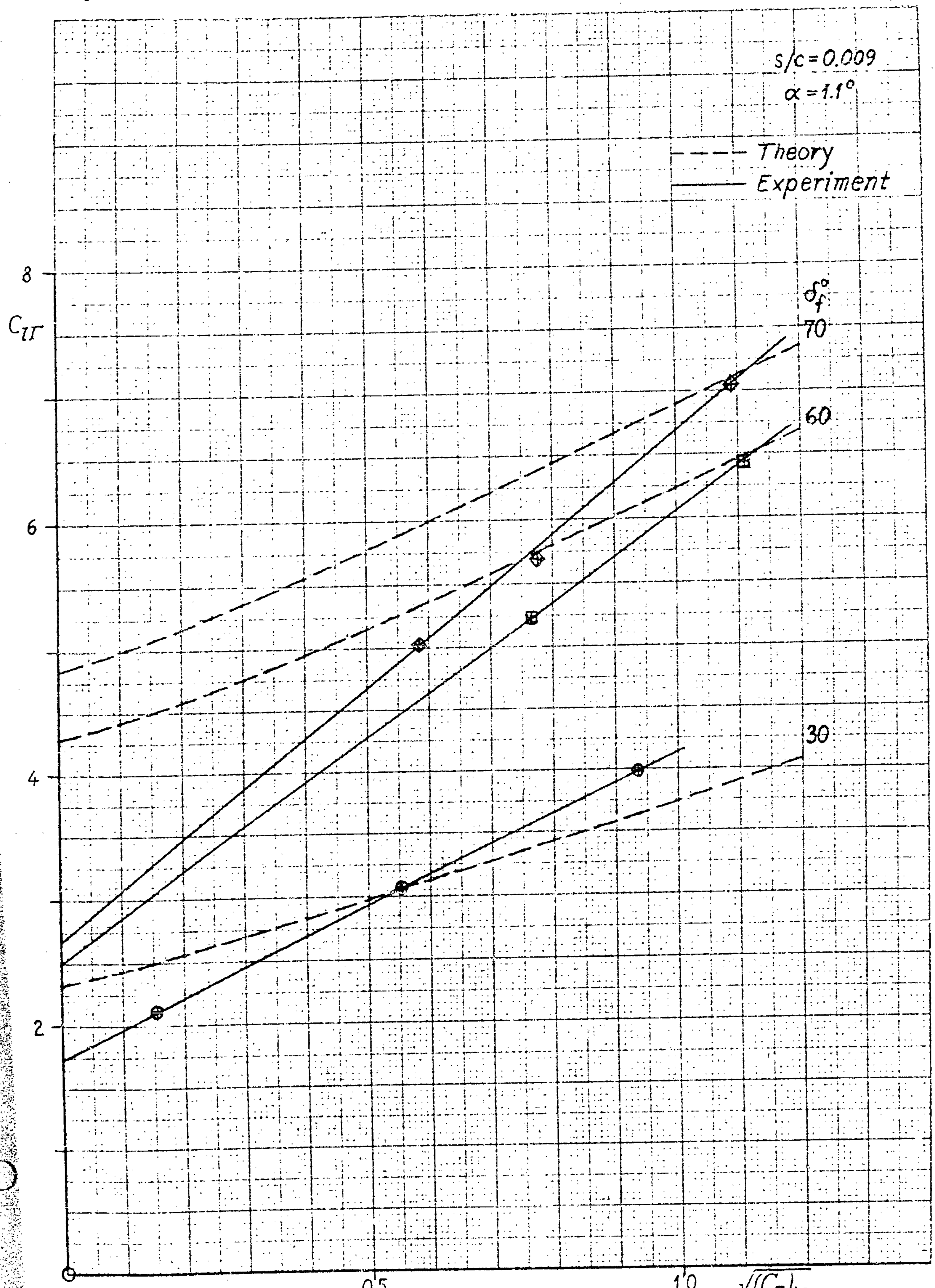

s/c=0.009
α=1.1°
Theory
Experiment
8
6
4
2
C_{LT}
δ_f°
70
60
30
0
0.5
1.0
$\sqrt{(C_E)_{te}}$

Fig.4

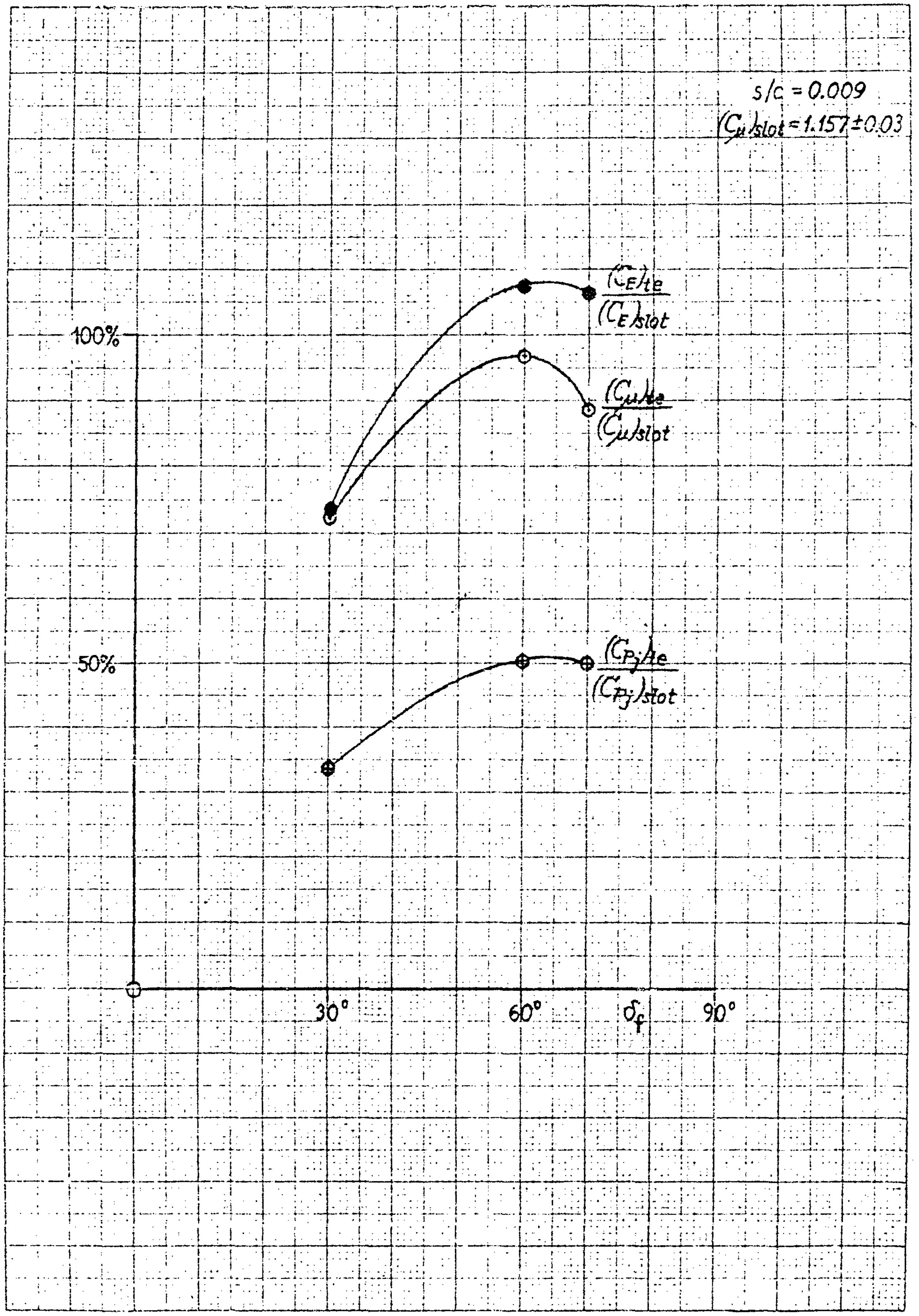

s/c = 0.009
$(C_\mu)_{slot} = 1.157 \pm 0.03$
$\frac{(C_E)_{te}}{(C_E)_{slot}}$
$\frac{(C_\mu)_{te}}{(C_\mu)_{slot}}$
$\frac{(C_{Pj})_{te}}{(C_{Pj})_{slot}}$
100%
50%
30°
60°
δ_f
90°

Fig.5

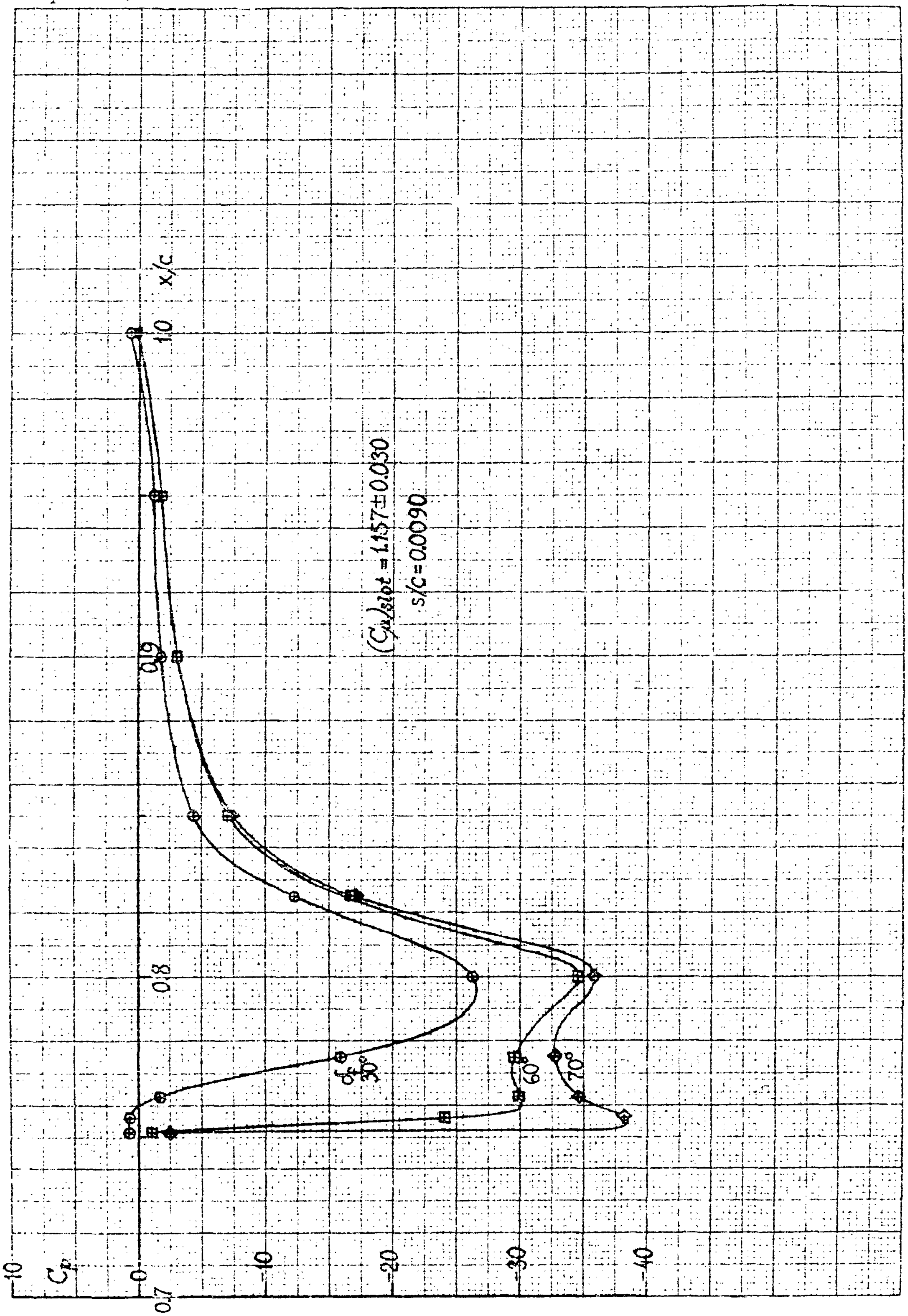

$(C_\mu)_{slot} = 1.157 \pm 0.030$
s/c = 0.0090
x/c
C_p
10
0
-10
-20
-30
-40
0.7
0.8
0.9
1.0
δ_f = 30°
60°
70°

Fig.6

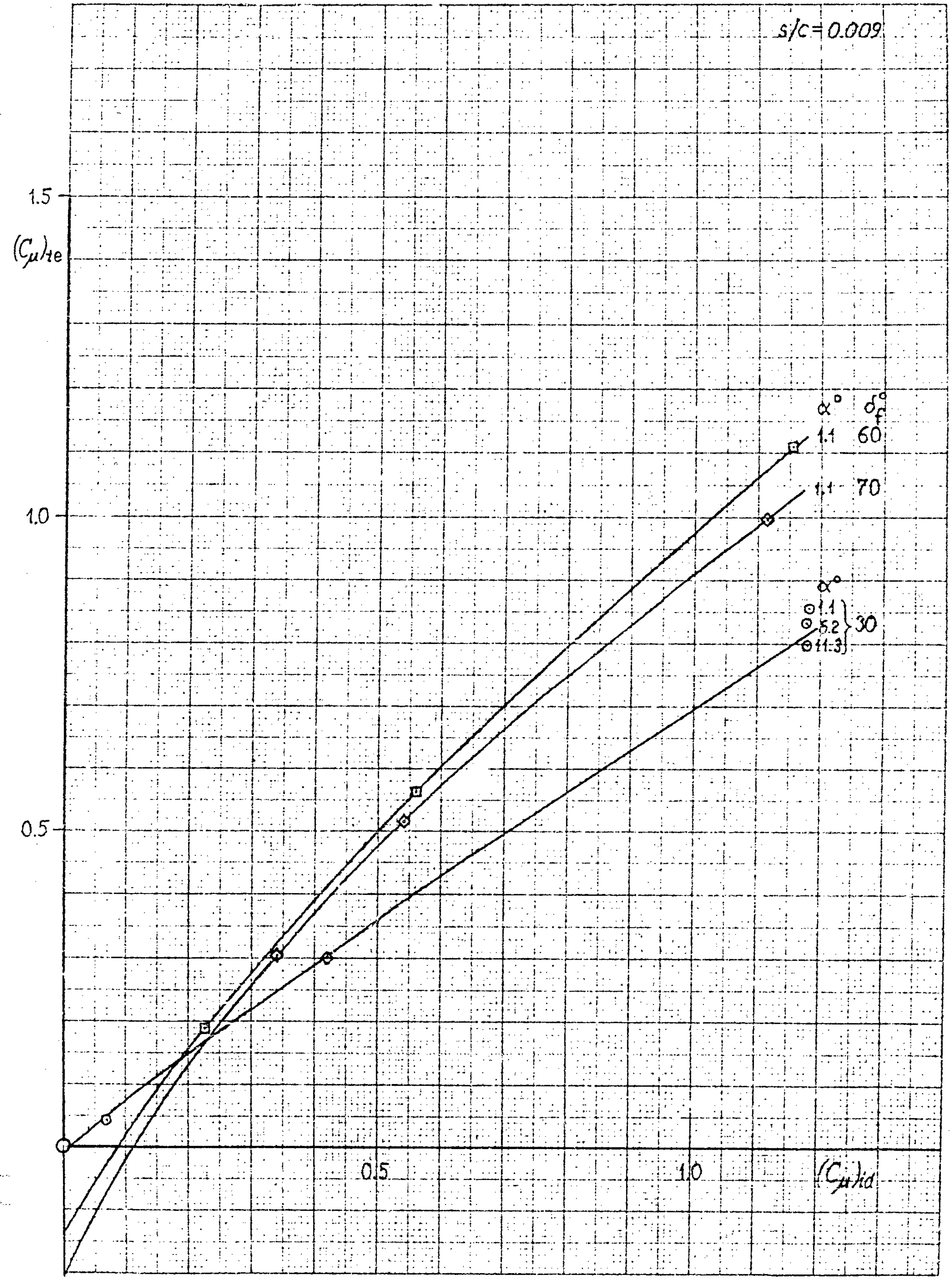

s/c = 0.009
1.5
1.0
0.5
$(C_\mu)_{te}$
0.5
1.0
$(C_\mu)_{id}$
α° δ_f°
1.1 60
1.1 70
α°
1.1
5.2
11.3
30

Fig.7

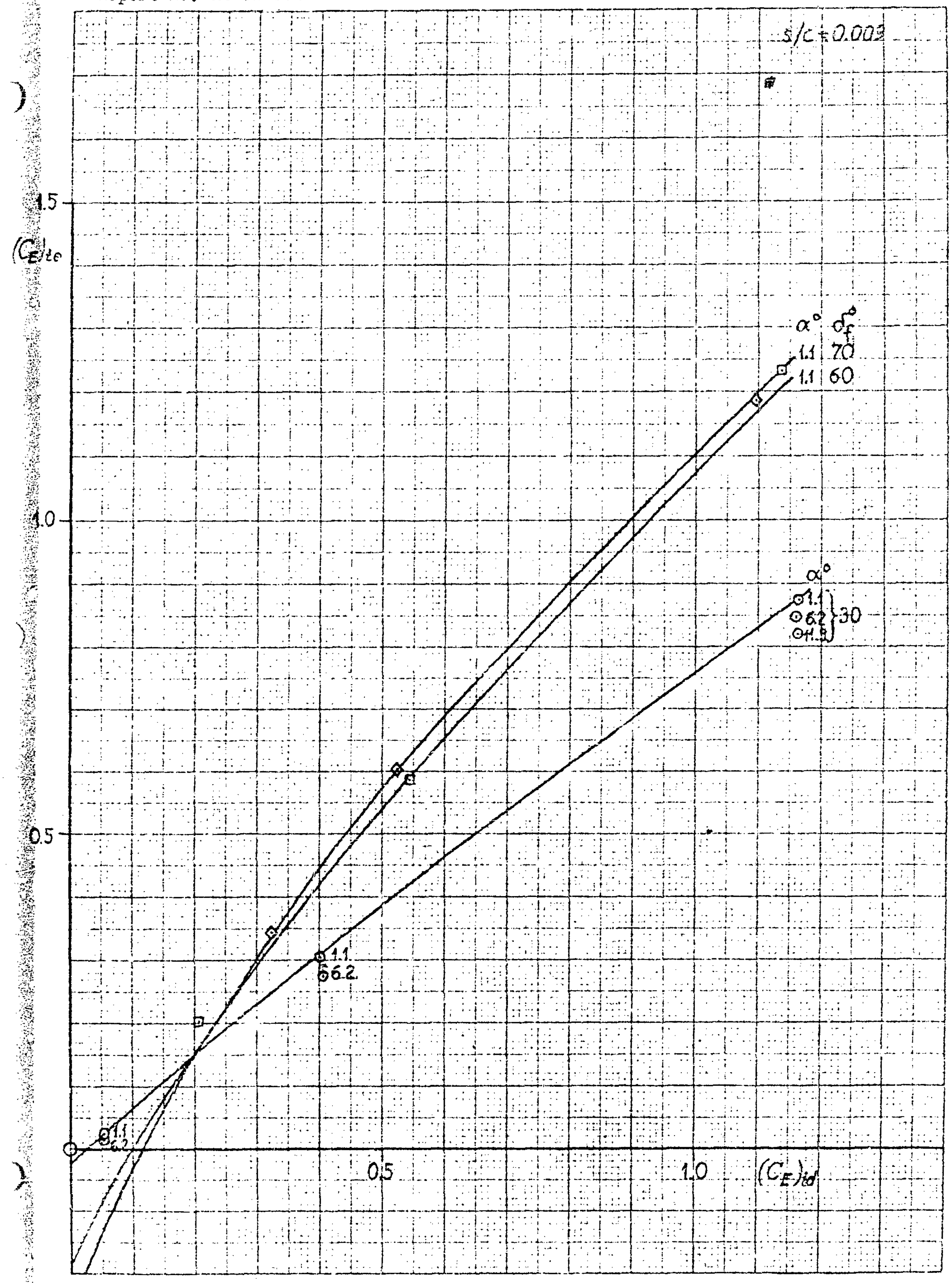

s/c = 0.003
1.5
$(C_E)_{te}$
1.0
0.5
0.5
1.0
$(C_E)_{id}$
α° δ°f
1.1 70
1.1 60
α°
1.1
6.2
11.8
30
1.1
6.2
1.1
6.2

Fig.8

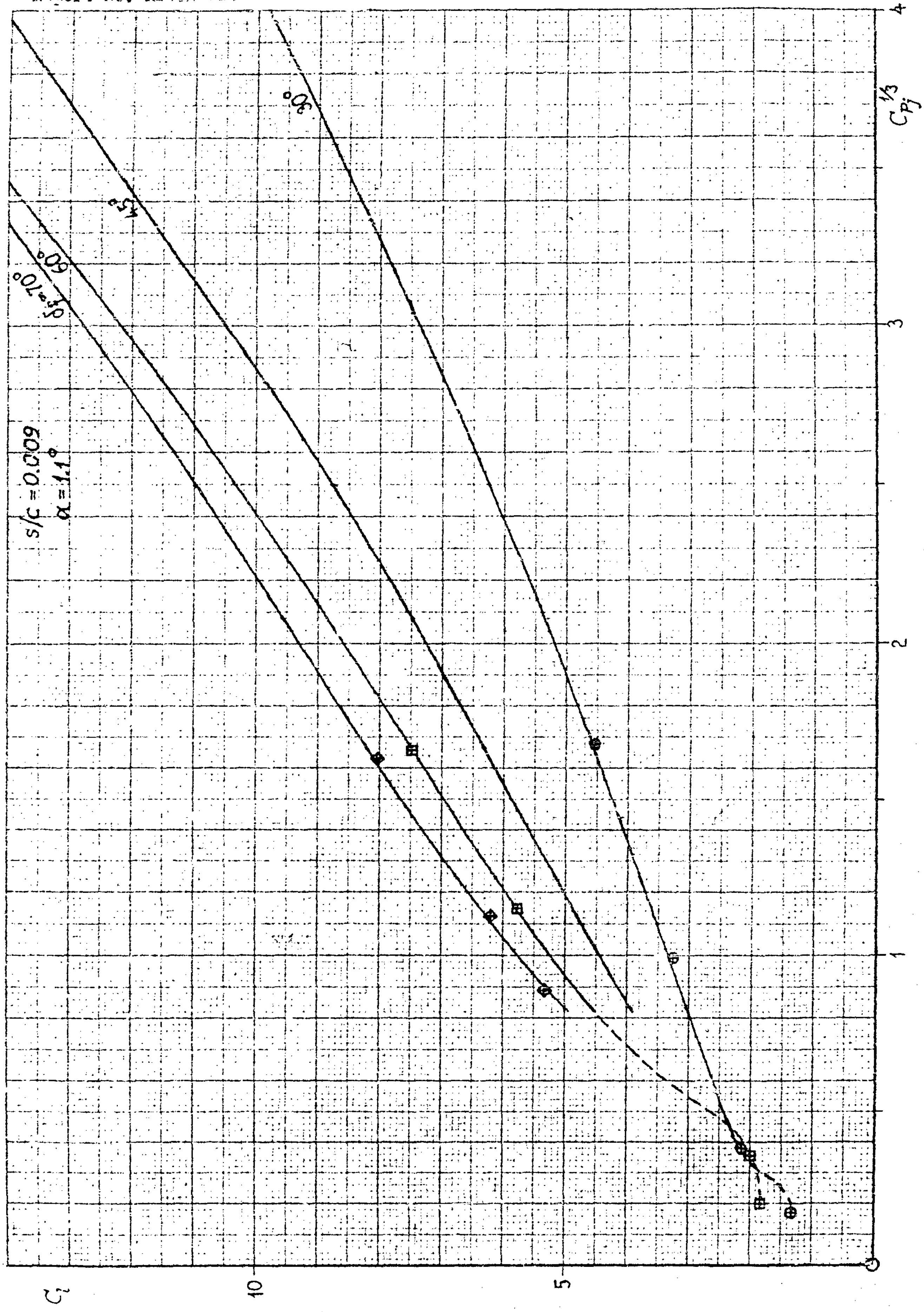
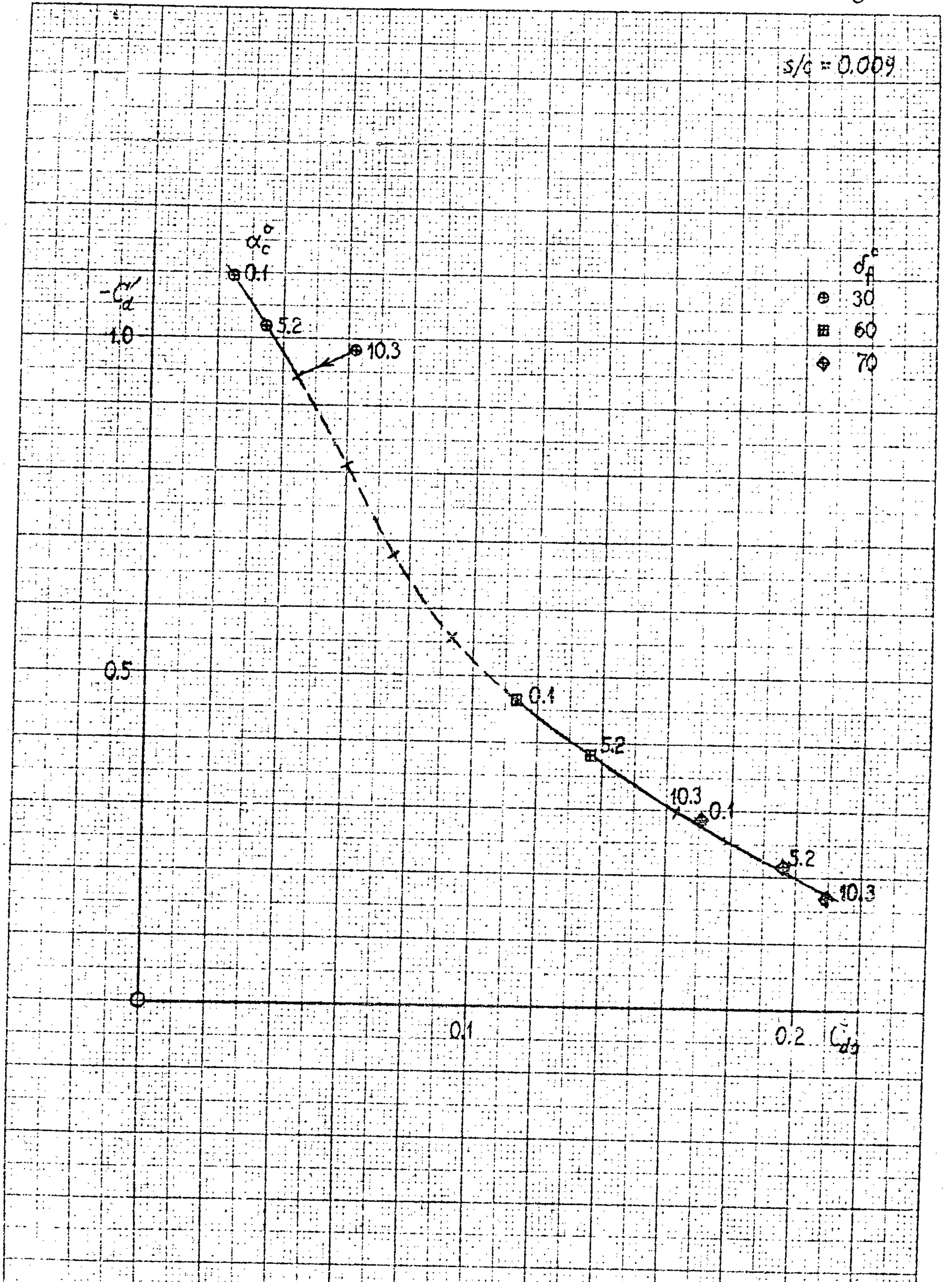


Fig. 9



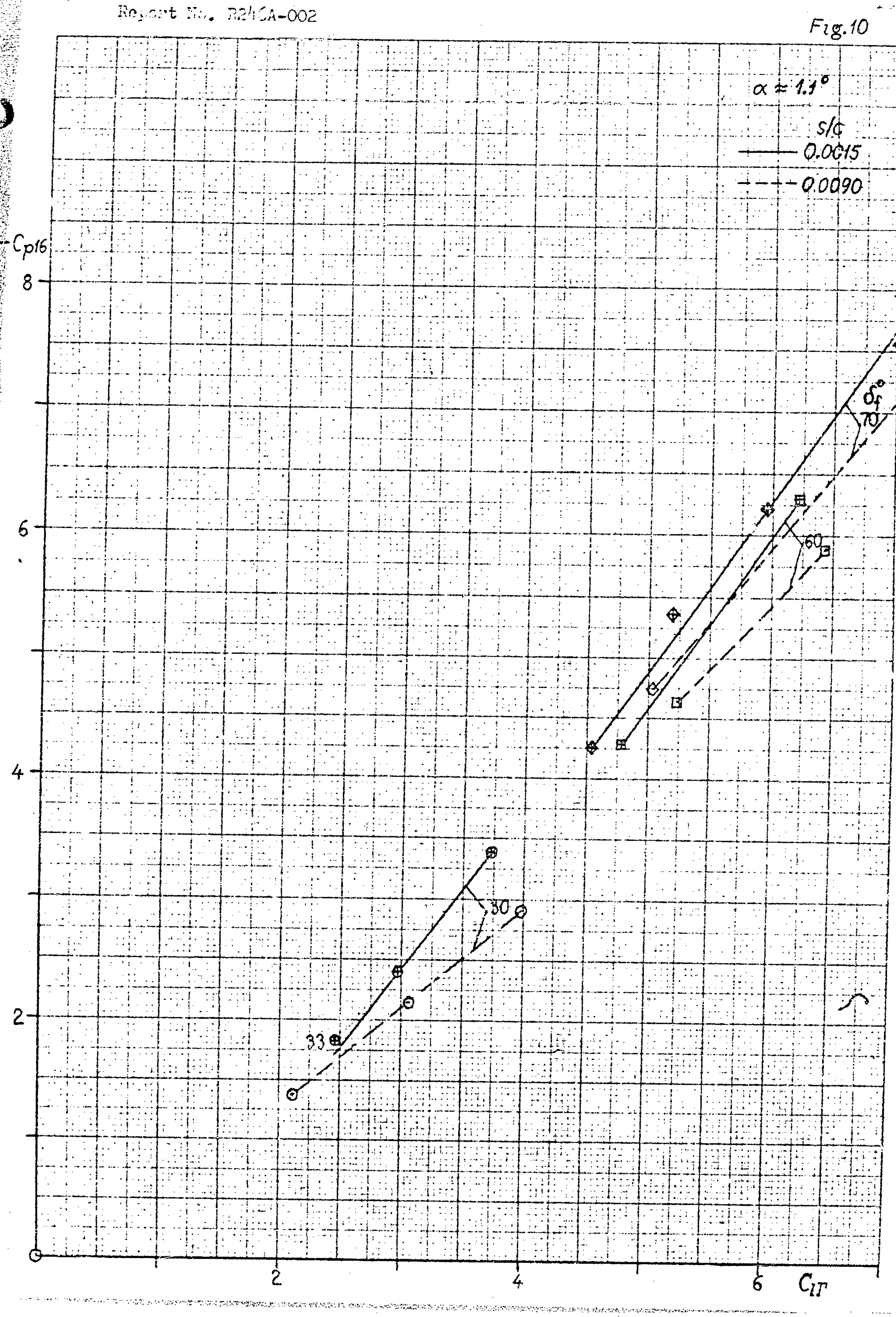
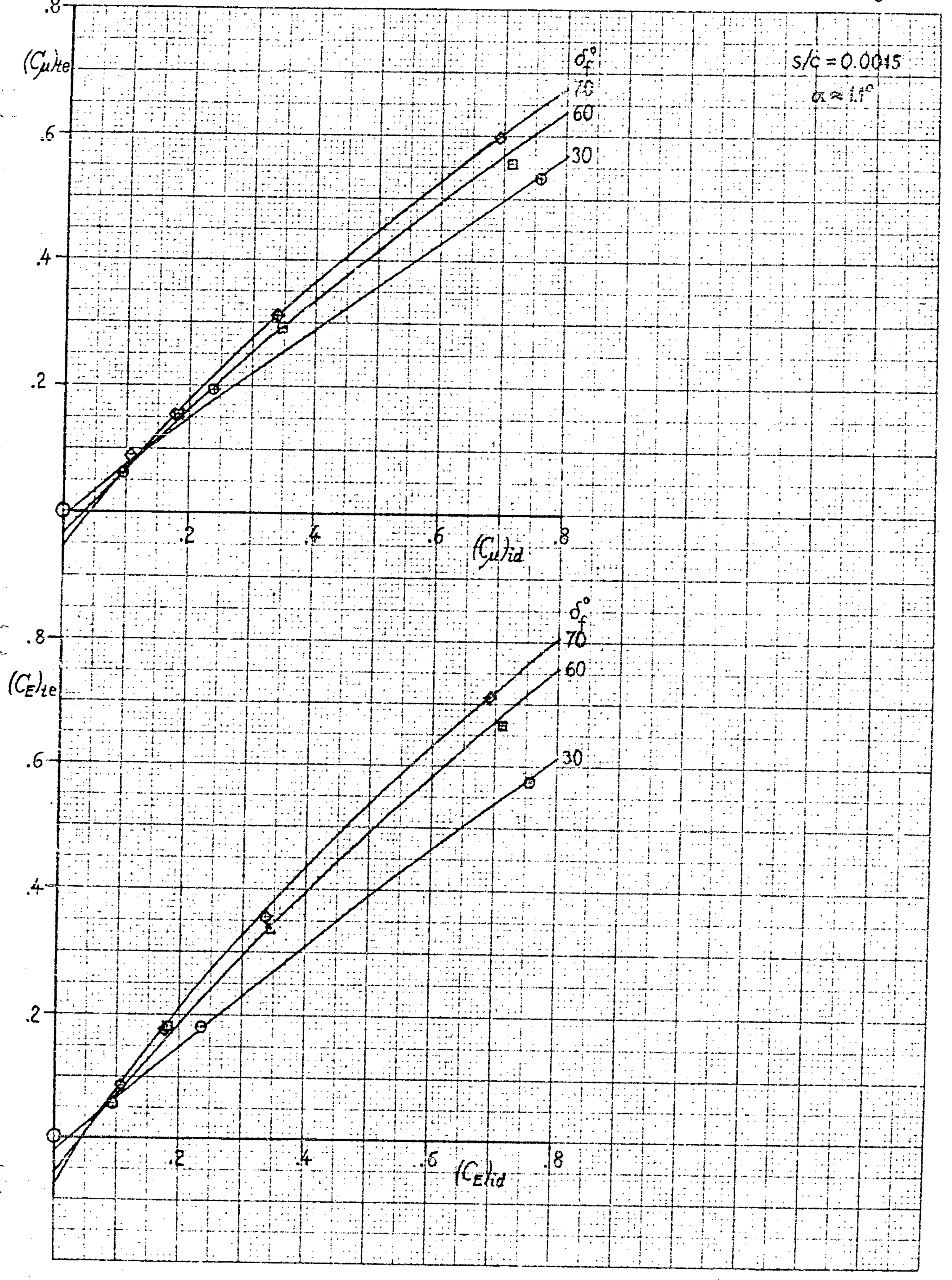


Fig.10
$\alpha \approx 1.1°$
s/c
0.0015
0.0090
$-C_{p16}$
8
6
4
2
$\delta_f°$
70
60
30
33
2
4
6
C_{IT}

Fig.11


.8
.6
.4
.2
$(C_\mu)_{te}$
δ_f°
70
60
30
s/c = 0.0015
$\alpha \approx 11^\circ$
.2
.4
.6
.8
$(C_\mu)_{id}$
.8
.6
.4
.2
$(C_E)_{te}$
δ_f°
70
60
30
.2
.4
.6
.8
$(C_E)_{id}$

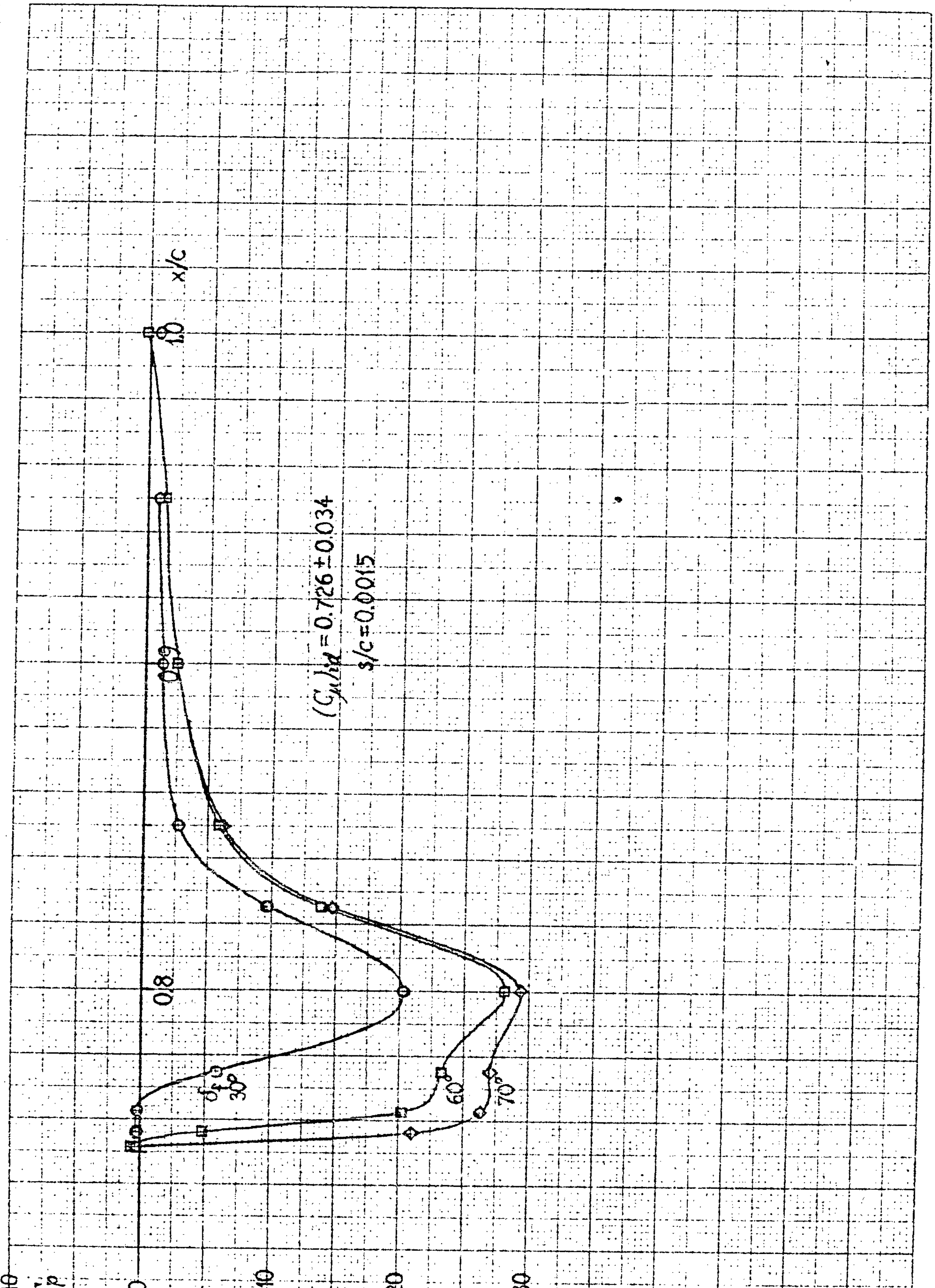
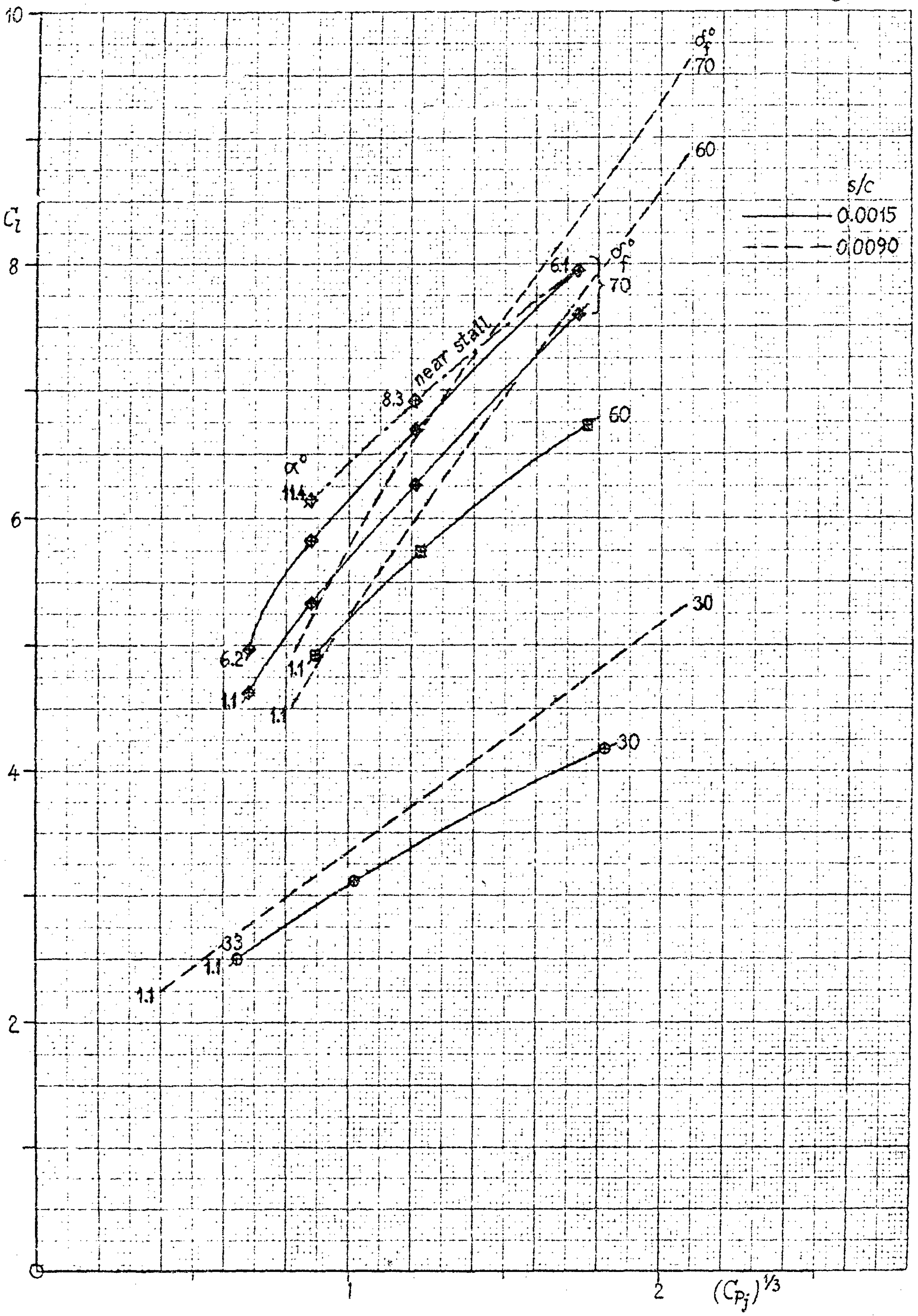

C_p
10
0
-10
-20
-30
0.7
0.8
0.9
1.0
x/c
$(C_\mu)_{jet} = 0.726 \pm 0.034$
s/c = 0.0015
δ_f
30°
60°
70°

Fig.13


10
8
6
4
2
0
1
2
C_l
$(C_{p_j})^{1/3}$
s/c
0.0015
0.0090
δ_f°
70
60
30
α°
near stall
11.4
8.3
6.1
6.2
1.1
3.3